Introduction to Superconductivity
Exercise Sheet 6

Discussion date: 28-October-2015

Exercise 1: The Little-Parks experiment: Fluxoid quantization and its effect on T_c.

In the lecture you have already learned about basic flux quantization. The Little-Parks experiment goes further and considers the fluxoid quantization *to explain how T_c depends on the applied field.*

We consider a superconducting film of thickness $d \ll \lambda, \xi$ in the form of a hollow cylinder of radius $R \gg d$ in a magnetic field H applied along the axis of the cylinder.

(a) What are the effects of $d \ll \xi$ and $d \ll \lambda$ on the amplitude of the order parameter $|\Psi|$ and the supercurrent j_s?

(b) The second Ginzburg Landau equation can be written as (using Gaussian units)

$$\boldsymbol{j_s} = \frac{e^*}{m^*}|\Psi|^2 \left(\hbar\nabla\varphi - \frac{e^*}{c}\boldsymbol{A}\right) = e^*|\Psi|^2\boldsymbol{v_s}, \tag{1}$$

where e^* and m^* are the charge and mass of the superconducting particles, i.e. $e^* = 2e$ and $m^* = 2m_e$. Write down the Ginzburg Landau free energy and express the gradient term using the supercurrent velocity v_s.

(c) Consider the path integral of the supercurrent velocity around the cylinder. Find an expression for the supercurrent velocity v_s in terms of the total flux Φ and the flux quantum $\Phi_0 = \frac{hc}{2e}$.

Comment: You will find that not the flux alone is quantized, but that it is the so-called fluxoid *which can take only discrete values.*

(d) While the fluxoid is quantized and can thus take different values, the actual configuration is the one with minimal free energy. For which v_s is this the case? Sketch v_s against Φ/Φ_0.

(e) What is the effect of the fluxoid quantization on the order parameter $|\Psi|^2$? Find the value of the amplitude of the order parameter by minimizing the free energy and express the result using $|\Psi_0|$ and ξ.

The critical temperature is defined through $|\Psi|^2 = 0$. What is therefore the effect on T_c? Sketch T_c against Φ/Φ_0. Or, as was done in the original paper by Little and Parks, sketch a H vs T phase diagram, indicating the superconducting and the normal region.

(f) What is the period of the oscillations (of the magnetic field) for a cylinder of radius $R = 7000$ Å?

(g) Have a look at the original paper (main text is 3 pages) and compare your sketches for the variation of the critical temperature to their experimental results (the 7 page file has an appendix of clearer figures):

W. A. Little, R. D. Parks, 'Observation of Quantum Periodicity in the Transition Temperature of a Superconducting Cylinder', Phys. Rev. Lett. 9, 9 (1962).