Introduction to String Theory — Problem Set 1

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1.1. On the importance of quantum gravity

Let us develop some intuition about orders of magnitude.

a) Consider a gravitational atom, that is an electron bound to a neutron by the gravitational interaction (neglect electromagnetic dipole effects). Perform a semiclassical calculation to determine the radius of the orbit of the electron (first Bohr radius). Relate this radius to a comparable distance in physics.

b) In *natural units* (where $\hbar$, G and c are set to 1), a stellar black hole radiates like a black body at a temperature given by

$$k_B T = \frac{1}{8\pi M}\,, \tag{1.1}$$

where k_B is the Boltzmann constant and M is the mass of the black hole. Give the temperature in SI units by reinserting G, $\hbar$ and c appropriately, then compute the temperature of a black hole with mass equal to one solar mass.

1.2. Classical motion of strings and oscillation modes

Consider a string with tension T_0 stretching along the x direction from $x = 0$ to $x = 2a$. The string oscillates along the transversal direction y, and the transversal displacement $y(x,t)$ satisfies the equation [1]

$$\ddot{y} - \frac{\mu(x)}{T_0}\,y'' = 0\,, \tag{1.2}$$

where we use the shorthand notation $\dot{y} := \frac{\partial y}{\partial t}$ and $y' := \frac{\partial y}{\partial x}$.

a) With the ansatz

$$y(x,t) = \psi(x)\,\sin(\omega t + \phi)\,, \tag{1.3}$$

derive the (ordinary) differential equation that the profile of the oscillation $\psi(x)$ has to satisfy. What is the physical meaning of eq. (1.3)?

b) Assume for now that $\mu(x) = \mu_0$ constant. Consider mixed Dirichlet-Neumann boundary conditions

$$y(0,t) = 0\,, \qquad y'(a,t) = 0\,.$$

Determine the allowed oscillation frequencies ω and the solution of the equation of motion for the string in this configuration. Use the ansatz of eq. (1.3).

c) Assume that $\mu(x) = \mu_1$ for $0 \le x < a$, $\mu(x) = \mu_2$ for $a \le x \le 2a$; this situation describes two strings joined at an endpoint. Consider now Dirichlet boundary conditions at the two endpoints, that is $y(0,t) = 0$, $y(2a,t) = 0$.

c.1) What boundary conditions should be imposed on $\psi(x), \psi'(x)$ at $x = a$?

c.2) Determine the conditions that the allowed oscillation frequencies must satisfy.

c.2) Calculate the lowest frequency of oscillation in the case of $\mu_1 = 3\mu_2$.

[1] We label both the direction and the displacement of the string with y, hoping not to generate confusion.