quantum melting
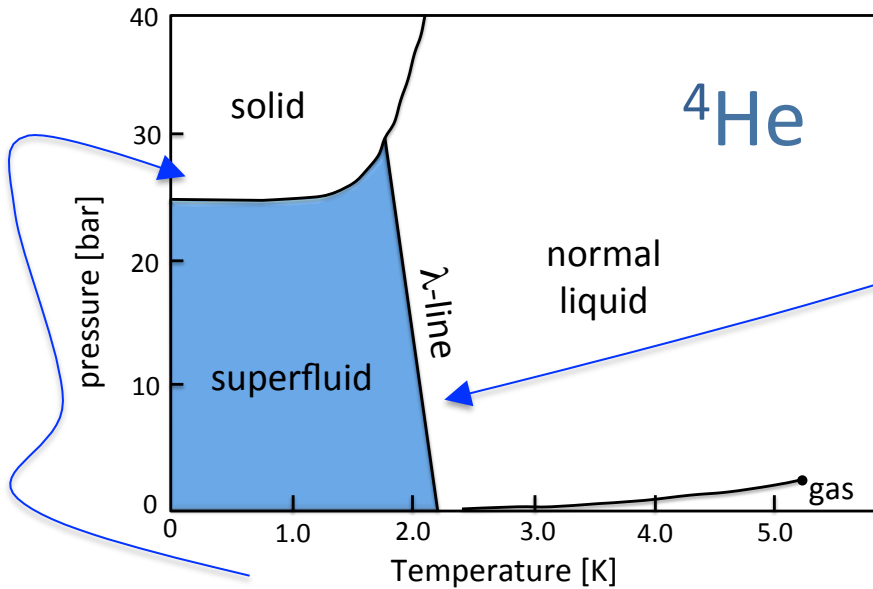
Superfluidity:

Bose-Einstein condensation
+
interaction
(collective modes)

^{4}He

solid

normal liquid

λ-line

superfluid

gas

pressure [bar]

Temperature [K]

C

$\dfrac{3k_B}{2}$

Phonons

T^3

real bose liquid

T_λ

T

C

$\dfrac{3k_B}{2}$

$T^{3/2}$

ideal bose gas

T_c

T
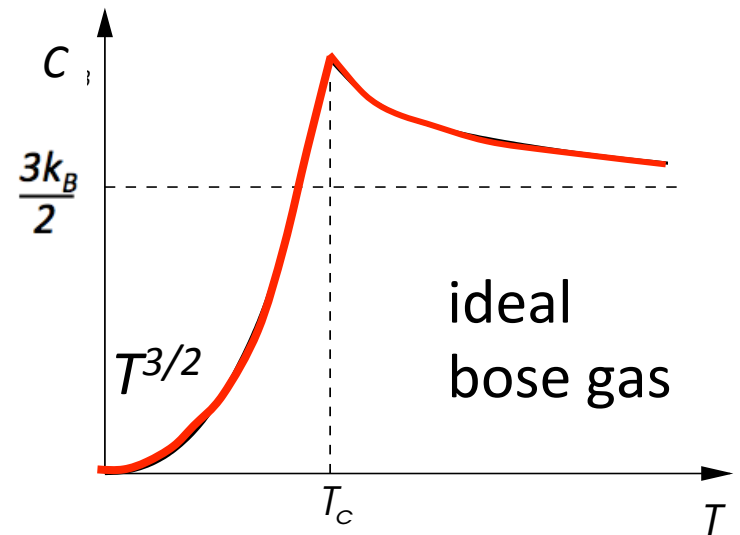
elementary excitations / collective modes

real spectrum

$\epsilon(p)$

roton

phonon

$v_c p$

Δ

p_0

p

energy

$= \sqrt{\epsilon_{\vec{k}}^2 + 2Un_0\epsilon_{\vec{k}}}$

Un_0

$E_{\vec{k}}$

$v\hbar k$

phonon

$\epsilon_{\vec{k}} = \dfrac{\hbar^2 k^2}{2m}$

$|\vec{k}|$
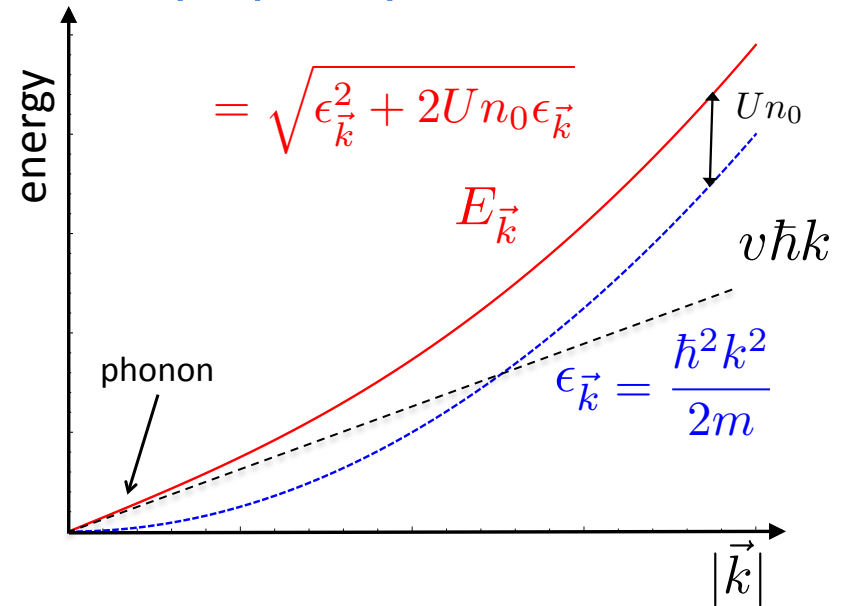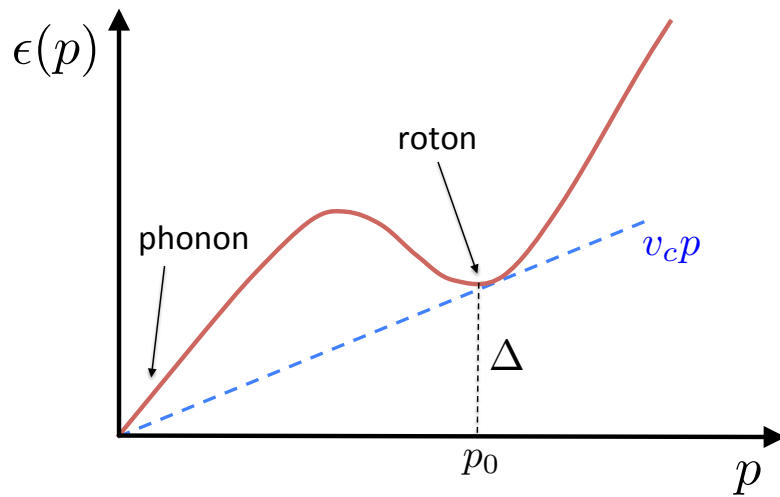
finite critical velocity

$v < v_c$	frictionless flow	no decay possible
$v > v_c$	slowing down	decay into elementary excitations

depletion of condensate at *T=0*

$n_0 < n$

n_0

condensate

n_k

k

not all bosons enter the condensate
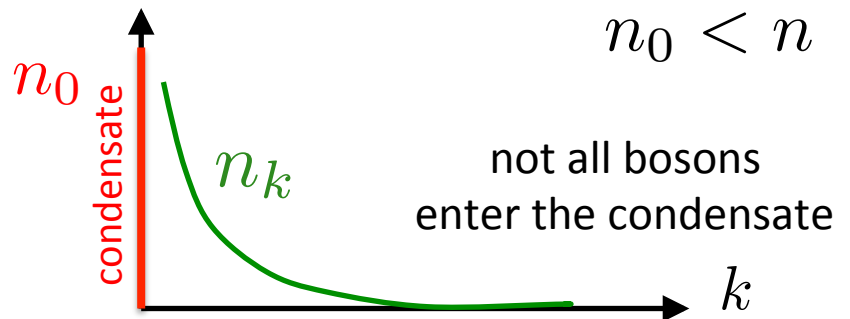
Gross-Pitaevskii equation

$$\hat{\Psi}(\vec{r}) = \psi_0(\vec{r}) + \delta\hat{\Psi}(\vec{r})$$

$$\psi_0(\vec{r}) = \sqrt{n_0(\vec{r})}\, e^{i\phi(\vec{r})}$$

condensate
wave function

contact interaction

$$\hbar\frac{\partial}{\partial t}\psi_0(\vec{r},t) = \left(-\frac{\hbar^2\vec{\nabla}^2}{2m} + V(\vec{r}) - \mu + U|\psi_0(\vec{r},t)|^2\right)\psi_0(\vec{r},t)$$

$$E = \int d^3r \left[\frac{\hbar^2}{2m}|\vec{\nabla}\psi_0(\vec{r})|^2 + \{V(\vec{r}) - \mu\}|\psi_0(\vec{r})|^2 + \frac{U}{2}|\psi_0(\vec{r})|^4\right]$$

$$\left.\begin{array}{l} V(\vec{r}) = 0 \\[1em] \mu = Un_0 \end{array}\right\} \quad \Rightarrow \quad |\psi_0|^2 = n_0$$
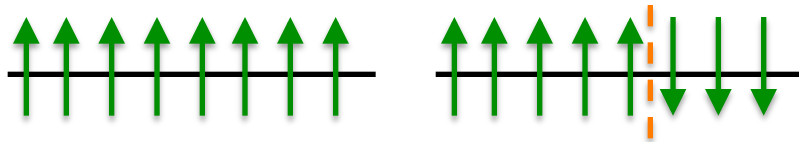
Berezinskii-Kosterlitz-Thouless transition

correlation function
in a 2D film

$$g(R) \propto \begin{cases} R^{-1/2}e^{-k_0 R} & \text{high temperature} \\ \\ R^{-\eta(T)} & \text{low temperature} \end{cases}$$

no long-range order at finite T, but change of correlation: exponential ⟷ algebraic

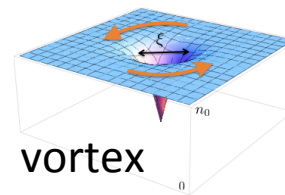## key element: topological excitation

1D Ising ferromagnet

domain wall

$$\Delta F = E_{\text{DW}} - TS_{DW}$$

$$= 2Js^2 - k_B T \ln N$$

no phase transition at *T > 0*

TD limit

$$\downarrow N \to \infty$$

entropy
dominates

2D superfluid film

vortex

$$E, S \propto \ln\left(\frac{R}{\xi}\right)$$

$$\Delta F = \left\{\frac{\hbar^2 n_0 \pi}{m} - 2k_B T\right\} \ln\left(\frac{R}{\xi}\right)$$

BKT-transition $$k_B T_{KT} = \frac{\hbar^2 n_0 \pi}{2m}$$