particle density

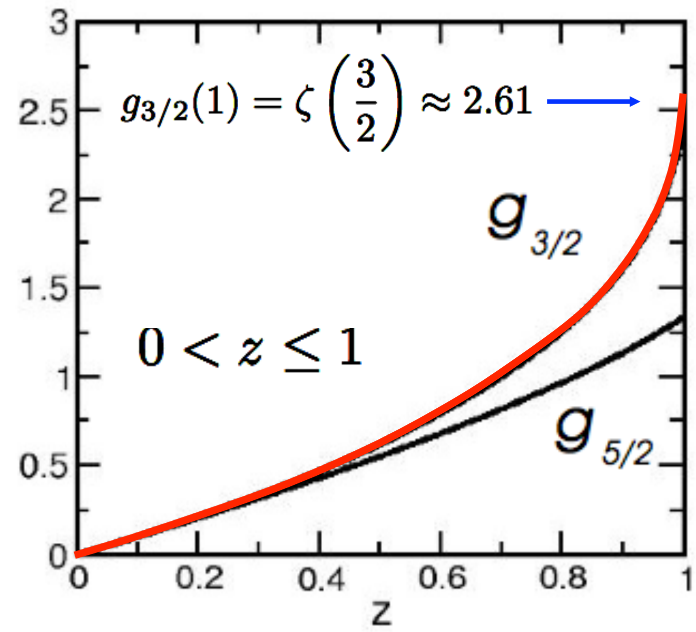
$$n = \frac{1}{\lambda^3} g_{3/2}(z)$$

$$g_n(z) = \sum_{l=1}^{\infty} \frac{z^l}{l^n}$$

thermal wavelength

$$\lambda = \frac{h}{\sqrt{2\pi m k_B T}}$$

$$T \searrow \;\Rightarrow\; \begin{cases} \lambda \nearrow \\ z \to 1 \end{cases}$$



critical point reached for

$$z = 1$$

$$\frac{N}{V}\lambda^3 = g_{3/2}(1) = \zeta(3/2)$$

$$T_c = \frac{h^2}{2\pi k_B m [\zeta(3/2) V/N]^{2/3}}$$

$$V_c = \frac{N}{\zeta(3/2)} \frac{h^3}{(2\pi m k_B T)^{3/2}}$$

fixed

N, V

N, T

particle density

$$n = \frac{1}{\lambda^3} g_{3/2}(z) \quad \Rightarrow \quad n = \frac{1}{\lambda^3} g_{3/2}(z) + n_0(T) = n_n(T) + n_0(T)$$

"normal" "condensed"

density of *p=0*-Bosons

state with *p=0*

not properly counted

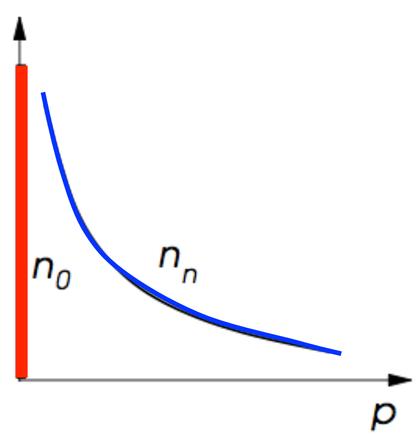
$$z < 1 \Rightarrow n_0 = 0$$

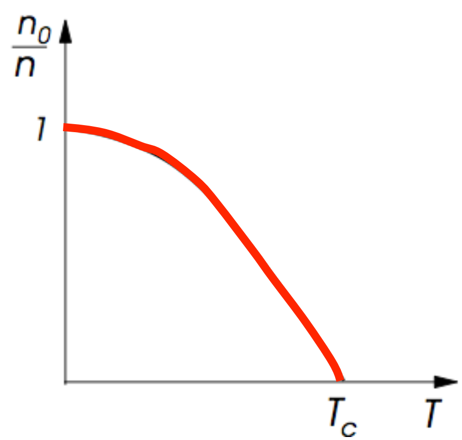
$$z = 1 \Rightarrow n\lambda_c^3 = \zeta(3/2)$$

$$n = \frac{\zeta(3/2)}{\lambda^3} + n_0$$

$$= n\frac{\lambda_c^3}{\lambda^3} + n_0$$

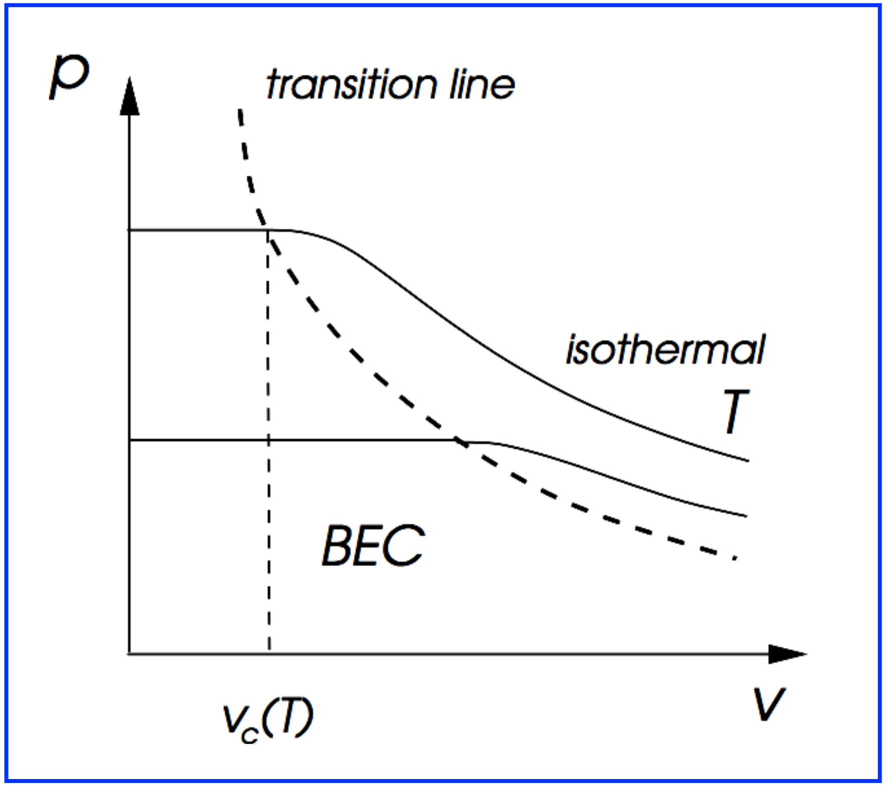
$$n_0(T)] = n\left[1 - \left(\frac{T}{T_c}\right)^{3/2}\right]$$





equation of state

$$p = \begin{cases} \frac{k_B T}{\lambda^3} g_{5/2}(z)\,, & V > V_c \\ \frac{k_B T}{\lambda^3} g_{5/2}(1)\,, & V < V_c \end{cases}$$



compressibility $V > V_c$

$$\kappa_T = \frac{N\lambda^6}{V k_B T g_{3/2}(z)^2} \frac{g'_{3/2}(z)}{g'_{5/2}(z)}$$

diverges at $z = 1$

entropy (fixed μ)

$$S(T,V,\mu) = -\left(\frac{\partial\Omega}{\partial T}\right)_{V,\mu} = \begin{cases} Nk_B\left(\frac{5v}{2\lambda^3}g_{5/2}(z) - \ln z\right), & T > T_c, \\ Nk_B\frac{5}{2}\frac{g_{5/2}(1)}{g_{3/2}(1)}\left(\frac{T}{T_c}\right)^{3/2}, & T < T_c \end{cases}$$

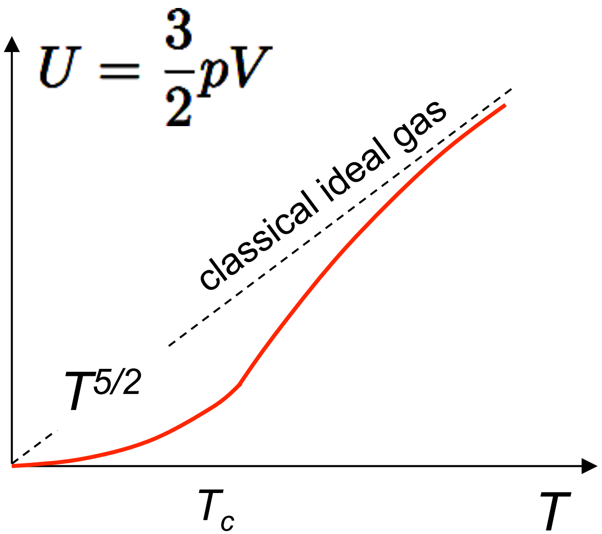
entropy per particle

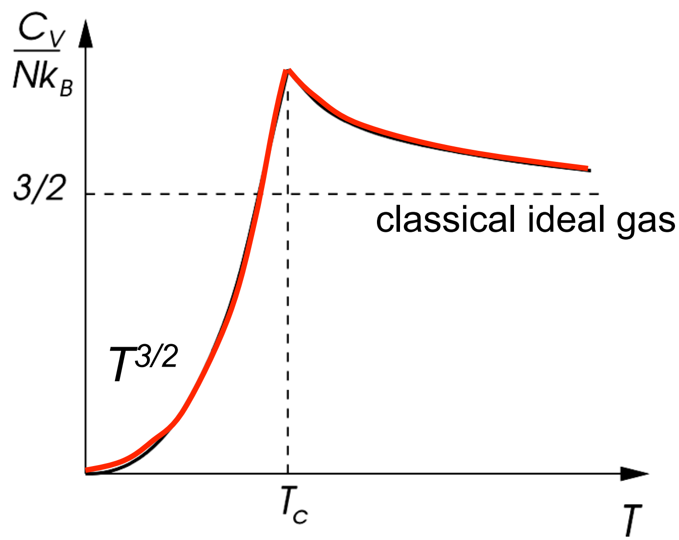
$$\frac{S}{N} = s\left(\frac{T}{T_c}\right)^{3/2} = \frac{n_n(T)}{n}s \qquad \text{with} \quad s = \frac{5}{2}k_B\frac{g_{5/2}(1)}{g_{3/2}(1)}$$

contribution to entropy from normal particles only

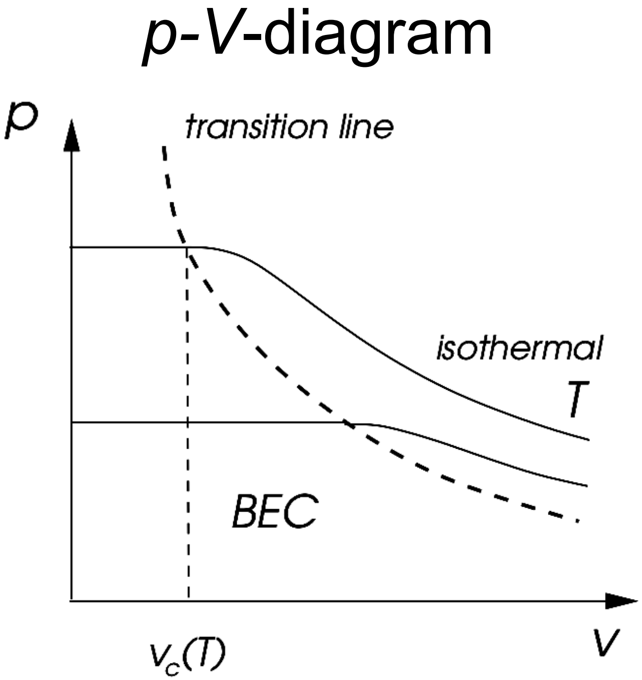
specific heat (fixed N)

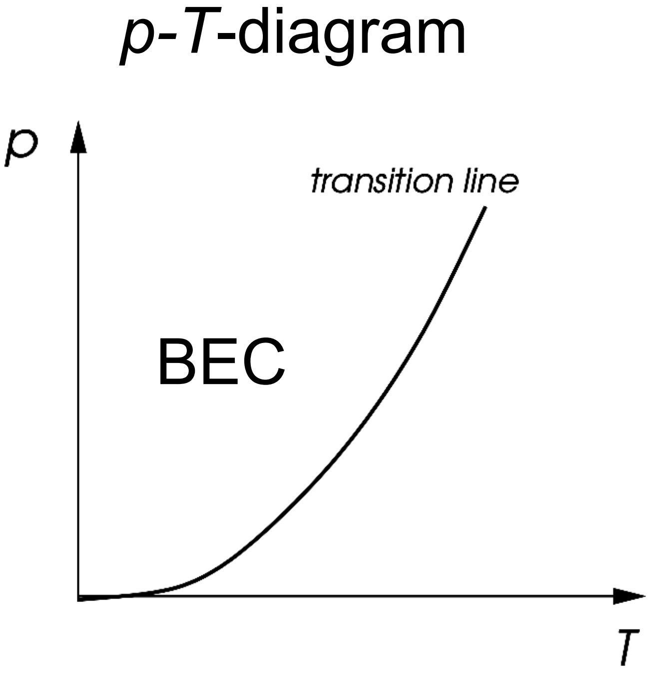
$$C_V = \left(\frac{\partial U}{\partial T}\right)_{V,N} = \begin{cases} N k_B \left(\dfrac{15 v}{4\lambda^3} g_{5/2}(z) - \dfrac{9}{4} \dfrac{g_{3/2}(z)}{g_{1/2}(z)} \right) \ , & T > T_c \ , \\ N k_B \dfrac{15}{4} \dfrac{g_{5/2}(1)}{g_{3/2}(1)} \left(\dfrac{T}{T_c} \right)^{3/2} \ , & T < T_c \end{cases}$$





phase diagram



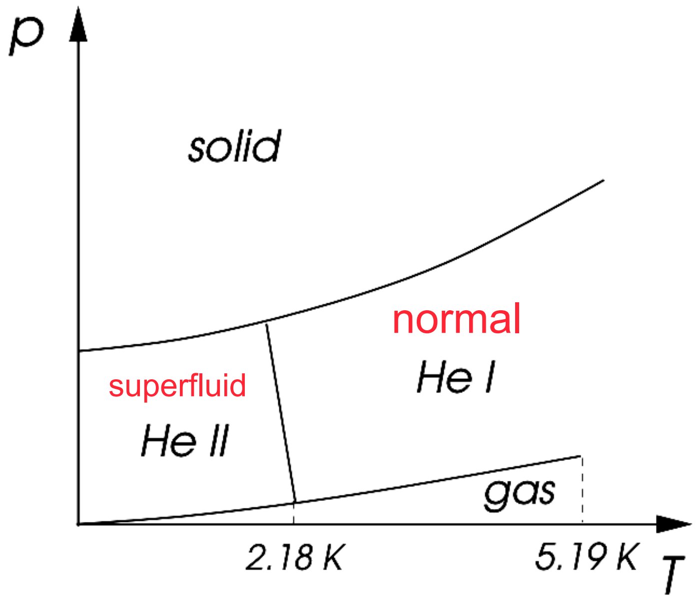


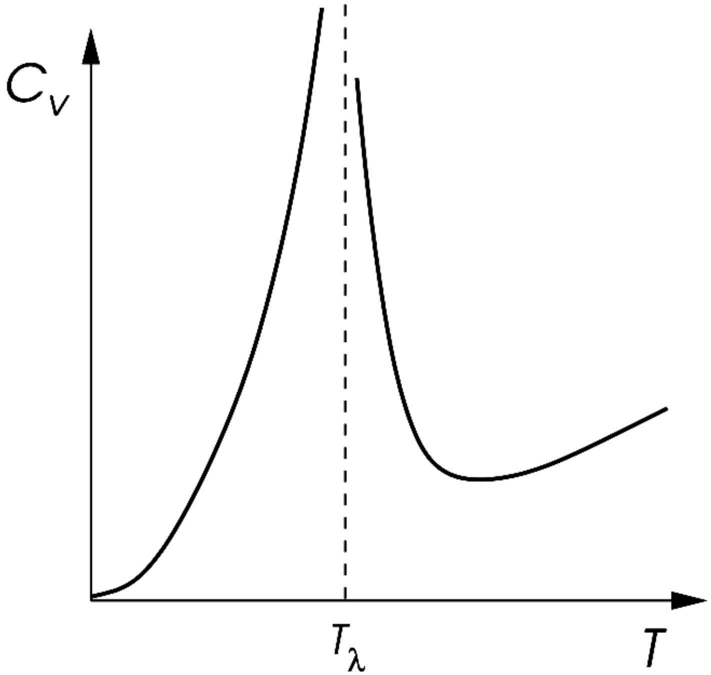
$$p_0 v^{5/3} = \frac{h^2}{2\pi m} \frac{g_{5/2}(1)}{[g_{3/2}(1)]^{5/3}}$$

$$p_0 = \frac{k_B T}{\lambda^3} g_{5/2}(1) \propto T^{5/2}$$

superfluid ^{4}He bosonic atoms

BEC ➡ supefluid ➡ frictionless flow
rigidity of condensate



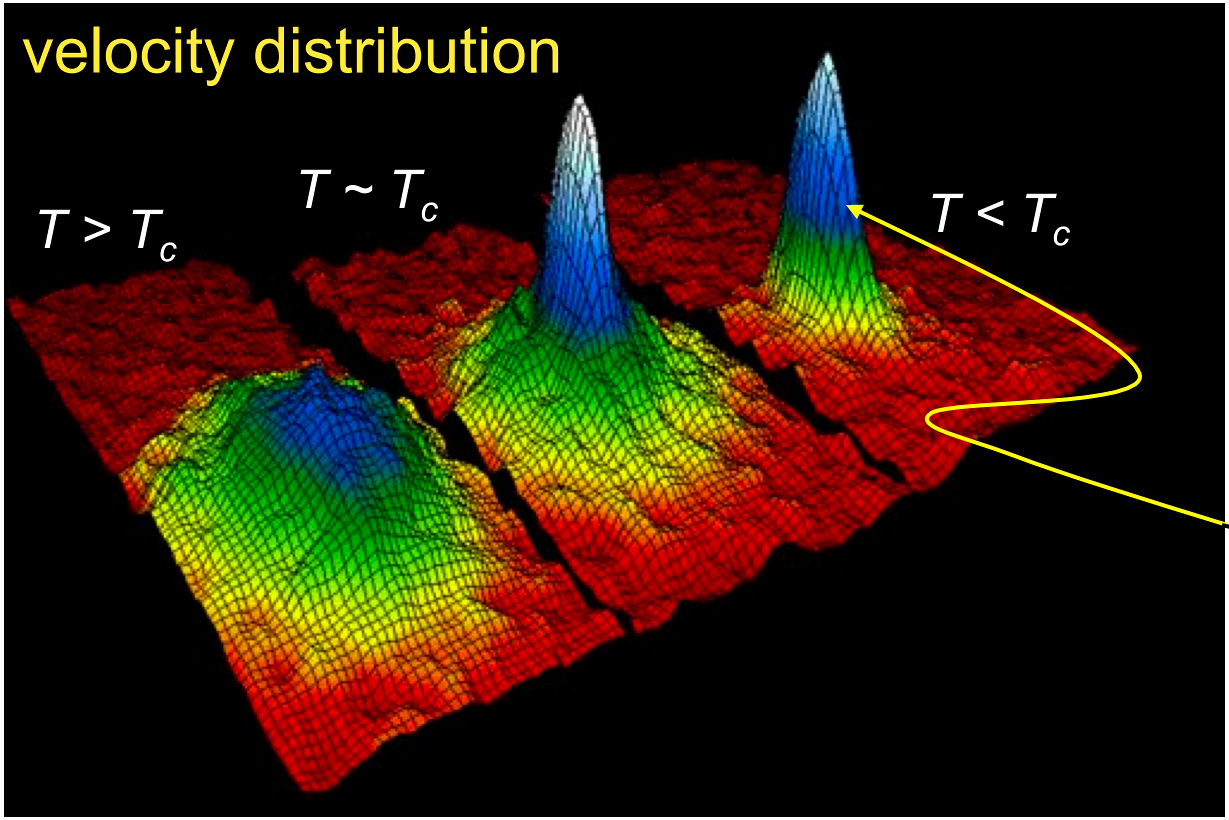


„ λ - transition “

ultra-cold atomic gases

^{87}Rb 37 electrons + 87 nucleons = 124 Fermions ➡ Boson

2000 Rb atoms in a trap $T_c = 170\ nK$



macroscopic occupation of state with $p = 0$