

Exercise 2.1 Classical Ideal Paramagnet

We consider an ideal paramagnet of magnetic moments in a magnetic field. The magnetic moments have only two orientations, parallel and antiparallel to the magnetic field. The Hamiltonian of the system is given by

$$\mathcal{H} = - \sum_{i=1}^N m_i H, \quad (1)$$

with $m_i = \pm m$, H as the magnetic field and N the number of magnetic moments.

- Calculate the internal energy, entropy, magnetization and magnetic susceptibility using the micro-canonical ensemble. *Hint:* Use combinatoric relations for binomial systems to determine the micro-canonical phase space count.
- Calculate the internal energy, entropy, magnetization and magnetic susceptibility using the canonical ensemble.

Exercise 2.2 Classical Ideal Lattice Gas

We consider N_1 particles on a lattice of N sites ($N = N_1 + N_2$), which have the condition that only one particle can occupy a site at a time. We assume that the particles have the energy E_A on N_1 sites and E_B on the other N_2 sites. Consider the situation that $N_1 < N_2$ and analyse the following situations in both the micro-canonical and grand-canonical ensemble.

- The energies satisfy $E_A < E_B$.
- The energies satisfy $E_A > E_B$.
- Vary the energies continuously between case a) and b).

Exercise 2.3 Classical Ideal Gas in a Harmonic Trap

We consider independent classical particles in a harmonic trap described by the Hamiltonian

$$\mathcal{H} = \sum_i \left\{ \frac{\vec{p}_i^2}{2m} + a\vec{r}_i^2 \right\}. \quad (2)$$

- Assume N particles and discuss the system in the micro-canonical ensemble.
- Assume N particles and discuss the system in the canonical ensemble.
- Assume a constant chemical potential μ and discuss the system in the grand canonical ensemble.

Note the differences. How would you determine/define compressibility?

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