Quantum computation with Josephson junctions

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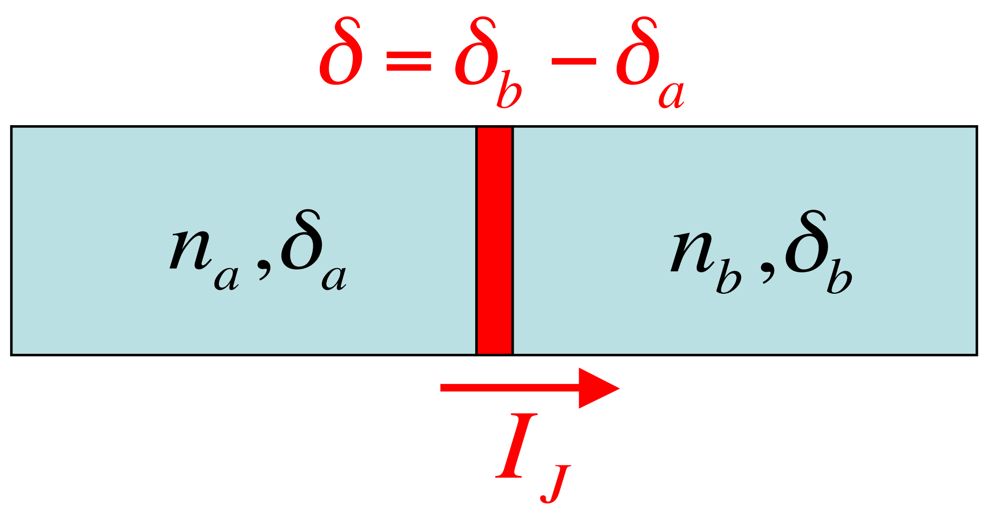
Follows closely
http://www.quantumlah.org/media/lectures/QT5201E-Haroche-Slides-5.pdf

Outline

- Josephson Hamiltonian
- Quantization of Josephson Hamiltonian
- Phase qubit controlled by current
- Phase qubit controlled by flux
- Capacitive coupling of qubits
- Inductive coupling of qubits

Isolated Josephson junction



- Two classical variables:
  - Particle number $2p = n_b - n_a$
  - Phase $\delta = \delta_b - \delta_a$
- Capacity:

$$V = \frac{Q}{C} = \frac{2ep}{C}$$

- Josephsons equations:
  - DC Josephsons effect

$$I_J = -2e\frac{dp}{dt} = I_0 \sin\delta$$

  - AC Josephsons effect

$$\frac{d\delta}{dt} = \frac{2eV}{\hbar} = \frac{4e^2p}{\hbar C}$$

- Looks like a pendulum!

Josephson Hamiltonian

$$H = E_C p^2 - E_J \cos\delta;\;\; E_C = \frac{2e^2}{C},\;\; E_J = \frac{\hbar I_0}{2e}$$

- Convince yourself by obtaining the equations of motion:

$$\frac{dp}{dt} = \frac{-1}{\hbar}\frac{\partial H}{\partial \delta} = -\frac{I_0}{2e}\sin\delta$$

$$\frac{d\delta}{dt} = \frac{1}{\hbar}\frac{\partial H}{\partial p} = \frac{4e^2 p}{\hbar C}$$

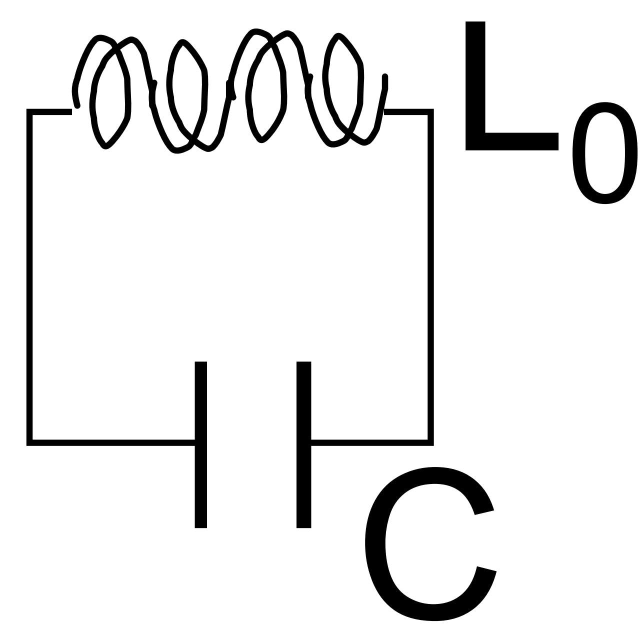
- p and δ are canonically conjugate variable!

Linearized version is a “lump circuit”

$$H_l = E_C p^2 + E_J \frac{\delta^2}{2}$$

E_C … capacitive energy

E_J … inductive energy



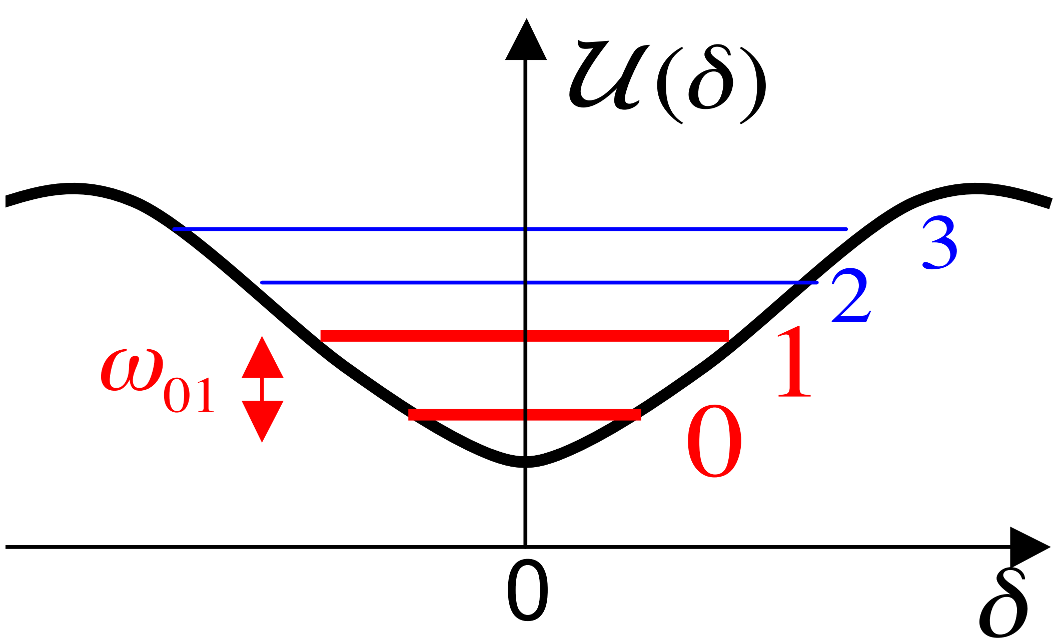
Quantizing the Josephson Hamiltonian

- Since p and δ are canonically conjugate we can postulate the commutator:

$$[p, \delta] = i$$

- Quantized energy levels (not harmonic!)

$$\omega_{01} \approx \sqrt{2E_J E_C}$$



- Particle Number (p) and phase (δ) cannot be measured exactly at the same time!
- Uncertainty relationship:

$$\langle \Delta p^2 \rangle = \sqrt{\frac{E_J}{8E_C}} \; , \quad \langle \Delta \delta^2 \rangle = \sqrt{\frac{E_C}{2E_J}}$$

- Two limiting cases:

$\frac{E_J}{E_C} \gg 1$ …phase well defined
-> phase qubit

$\frac{E_J}{E_C} \ll 1$ …charge well defined
-> charge qubit

Realising a superconducting Qubit

Prerequisites:

- controlled manipulation of qubit without disturbing adjacent elements
- controlled inter-qubit coupling
- detection of qubit state
- limited influence of external environment
- sufficiently long dephasing and decoherence times

Josephson Qubits:

- Qubit Hamiltonian adjustable by bias current and flux
- State preparation via rf-Pulses
- inter-qubit coupling achieved by capacitive or inductive coupling

Phase Qubit controlled by current

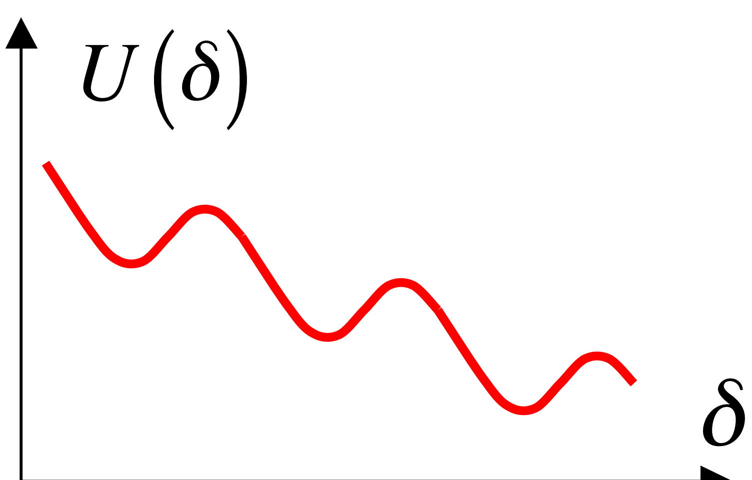
Josephson junction driven by constant current:

$$I = I_0 \sin\delta + \frac{dQ}{dt} = I_0 \sin\delta + 2e\frac{dp}{dt}$$

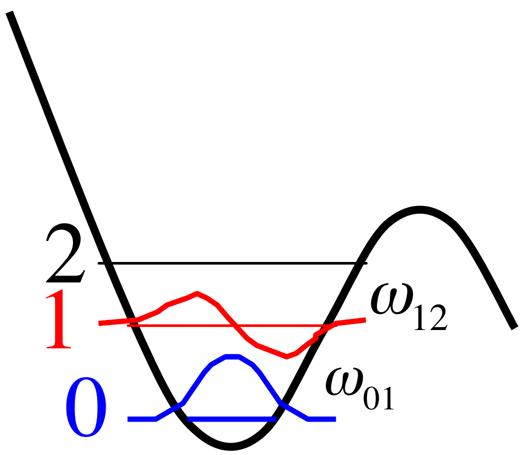
modified Hamiltonian:

$$H(I) = \frac{2e^2}{C}p^2 - \frac{\hbar}{2e}(I\delta + I_0\cos\delta)$$

washboard potential:

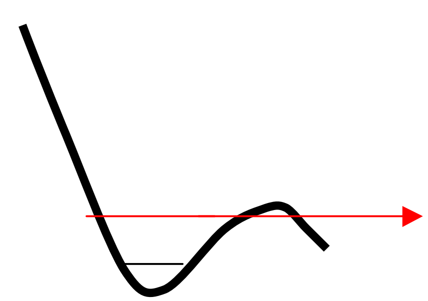


Frequency ω_{01} can be tuned by current I:

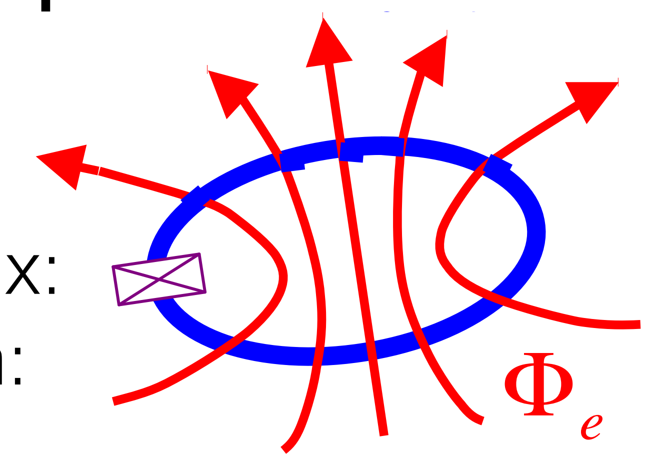


State detection:

- Increase I until the tunnel barrier is too low for state 1.
  - If state 1 -> critical value of $\frac{d\delta}{dt}$ exceeded -> normal metal state -> voltage drop



Phase qubit controlled by flux



Effect of the magnetic flux:

- Phase jump at junction:

$$\frac{2e}{\hbar}\oint \vec{A}\cdot d\vec{l} = 2\pi\frac{\Phi}{\Phi_0} = \delta \;;\; \Phi_0 = \frac{h}{2e}$$

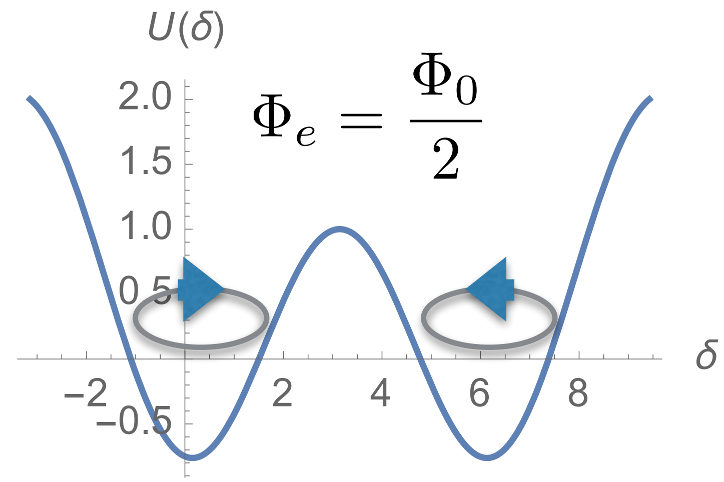
- Shielding current I

$$\Phi = \Phi_e - LI$$

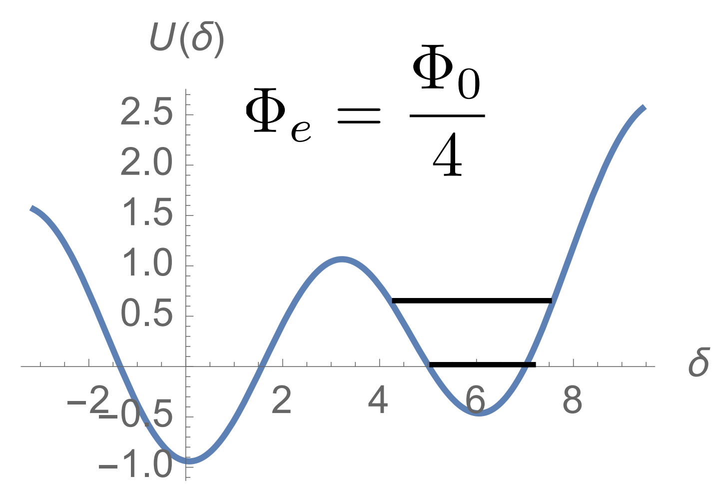
Add magnetic energy $\frac{1}{2}LI^2$ to the Hamiltonian:

$$H = \frac{2e^2}{C}p^2 + \frac{\Phi_0^2}{2L}\left(\frac{\delta}{2\pi} - \frac{\Phi_e}{\Phi_0}\right)^2 - \frac{\Phi_0}{2\pi}I_0\cos\delta$$

Symmetric case $\Phi_e = (n + \frac{1}{2})\Phi_0$, flux qubit:

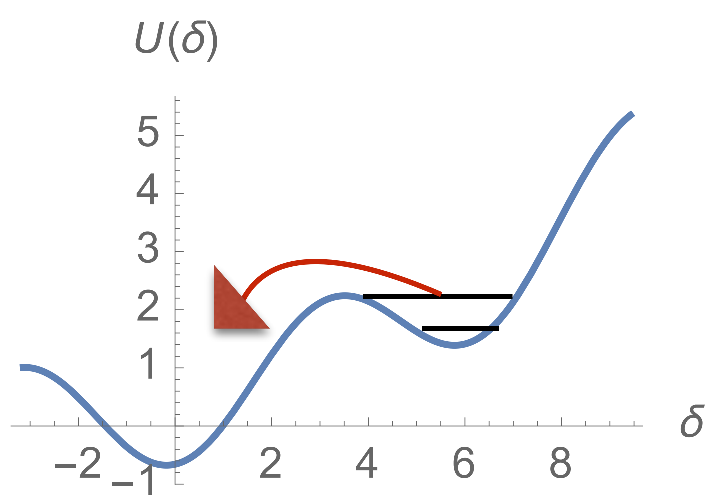


Otherwise asymmetric, phase qubit:



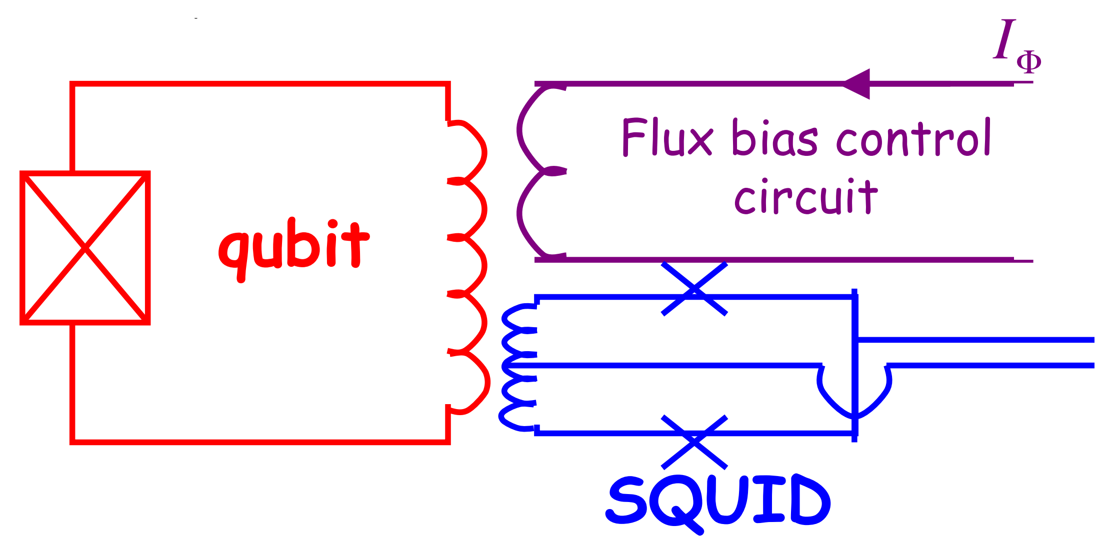
Detecting qubit state

Manipulation of flux Φ_e until qubit transits from one well to the other:



δ changes of order 2π
-> Φ_e changes of order Φ_0

Flux change detected by inductively coupled SQUID:



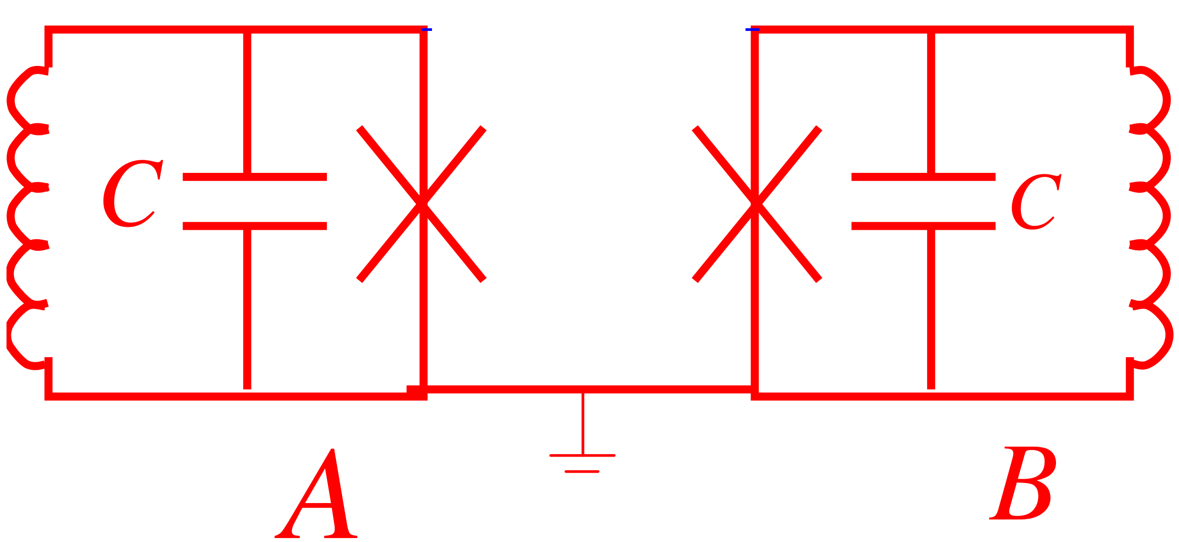
Manipulation of qubit state by an AC current with frequency close to ω_{01} :

$$H(t) = H_0 + \frac{\phi_0}{2\pi}\delta I_{rf}\sin(\omega t - \phi)$$

-> Rabi oscillations -> Rotation of qubit state!

Capacitive coupling of two phase qubits

Two identical phase qubits A and B coupled by $C_X \ll C$:



- Achieve tuneable coupling through frequency selection with bias current

Coupling energy:

$$H_{int} = \frac{1}{2} C_X (V_A - V_B)^2 = \frac{2e^2 C_X}{C^2} (p_A - p_B)^2$$

Regrouping terms:

$$H = H_A + H_B + H_{int} = H'_A + H'_B + H'_{int}$$

with

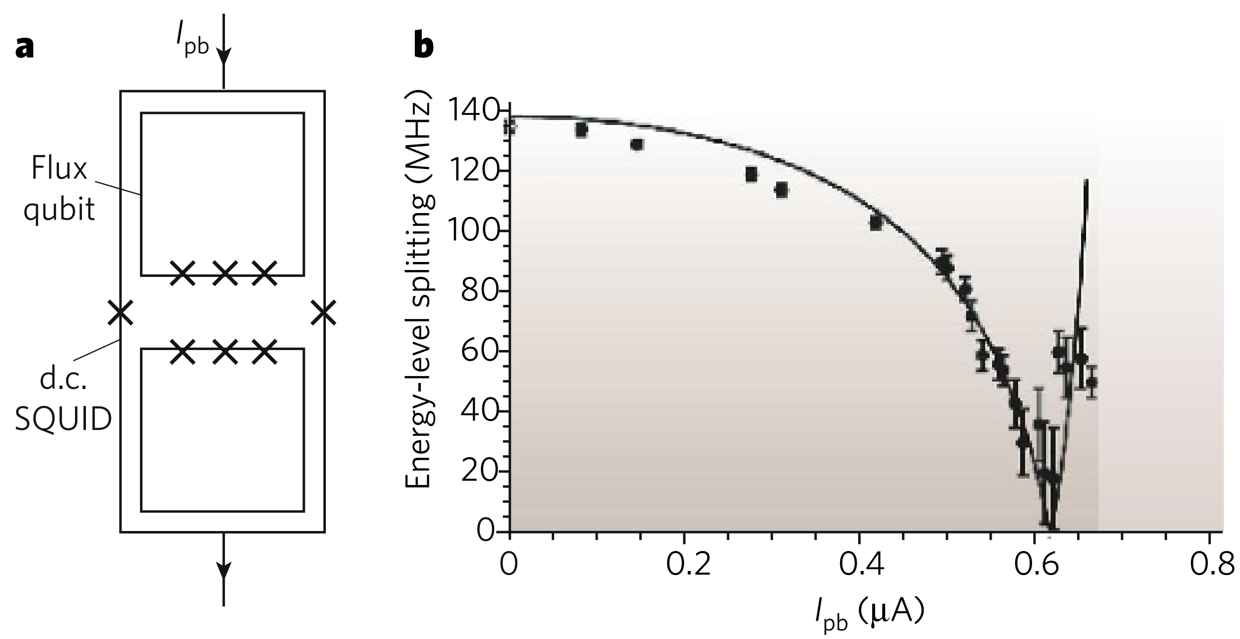
$$H'_i = \frac{2e(C + C_X)}{C^2} p_i^2 + U(\delta_i)$$

and $$H'_{int} = -\frac{4eC_X^2}{C^2} p_A p_B$$

Coupling of two flux qubits

For flux qubits the coupling can be achieved by the magnetic dipole-dipole interaction

- Can be coupled over long distances by “flux transformers”
  - Coupling through SQUID can be tuned



J.Clarke & F.Wilhelm, Superconducting quantum bits, Nature, 453, 1031 (2008)

Conclusions and Outlook

- charge and phase of Josephson junction are conjugate variables
- Frequency and detection of qubit achieved by current/flux bias
- Manipulation of qubit state by rf-pulses
- Coupled qubits can be used to create entanglement and realise quantum gates
- Decoherence can be modelled by complex impedance
- Coupling of qubit to rf LC Resonator -> Jaynes-Cummings Hamiltonian -> "Circuit QED”

Thank you for your attention.