ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Introduction to Superconductivity
Exercise Sheet 10

HS14
Dr. R. Chitra

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The last two exercise sheets will cover the BCS theory.

Exercise 1: BCS: Momentum distribution.

Here you derive in detail some results that were already stated in the lecture.

The momentum distribution of electrons in the BCS ground state is given by

$$N(k) = \sum_{k,\sigma} \langle c^\dagger_{k,\sigma} c_{k,\sigma} \rangle. \tag{1}$$

(a) Express $N(k)$

(i) at $T > 0$ and at $T = 0$ through v_k and u_k,

(ii) and at $T = 0$ through ξ_k and Δ.

(b) Calculate the relative fluctuations of the particle number in the BCS ground state

$$\frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle^2}, \tag{2}$$

by again expressing all the quantities first through v_k and u_k and then through ξ_k, Δ and ϵ_F.

Comment: You can set $\phi_k = 0 \pmod{2\pi}$, or neglect it, respectively. Why?

Exercise 3: BCS: Specific Heat at low Temperatures.

The specific heat is a very important quantity. Here we derive it in the BCS formalism.

Determine the specific heat C_s of the BCS superconductor as a function of T for $T \ll T_c$.