# Introduction to Superconductivity
# Exercise Sheet 3

HS14
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**Exercise 1: Superconducting sphere: Demagnetization factor and intermediate state.**

*The shape of superconducting objects is not negligible. When placed in an external background field, the actual magnetic field parallel to the surface of the sample may be locally enhanced. This can have an impact on the superconducting state even when the external field is below the critical field.*

We consider a superconducting sphere (type I) placed in a small external field $\boldsymbol{H}_{\text{ext}} = H_{\text{ext}}\hat{\boldsymbol{z}}$. We assume that $R \gg \lambda_L$ and neglect the screening effects near the surface such that $B = 0$ everywhere inside the sample.

(a) What is the total field? Formulate and solve this boundary-value problem.

*Comment: Look at the problem of a magnetic dipole in a uniform magnetic field. Plot the field distribution, and find the surface through which no flux passes.*

(b) What is the field parallel to the surface? What is it at the pole, and what at the equator?

(c) For which values of the external field $H_{\text{ext}}$ will there be an intermediate state of the sphere?

(d) What are the magnetic fields $B_{\text{equator}}$ and $B_{\text{pole}}$ inside the sphere in the intermediate state? Plot them together as a function of $H_{\text{ext}}$.

*Comment: Take $\boldsymbol{H}_{in}$ and $\boldsymbol{B}_{in}$ both parallel to the axis and constant, with $H_{in} = H_c$ and $B_{in} = B_0$, where $B_0$ has to be determined. What are the continuity conditions? What is now the expression for the field outside the sphere?*

(e) This effect can be measured with the so-called demagnetization factor, which is, however, only well defined for ellipsoids. It is given by

$$H_{\text{in}} = \frac{H_{\text{ext}}}{1-\eta}. \tag{1}$$

What is the demagnetization factor of a sphere? What is the general expression for ellipsoids for the range of the external field at which there will be an intermediate state?