Introduction to Superconductivity
Exercise Sheet 2

Discussion date: 1 October 2014

Exercise 1: Superconducting wires: Screening effects.

In the second lecture you learned about screening and the London penetration depth λ_L. With this additional knowledge, let us revisit Exercise 1.a) from Exercise Sheet 1.

How is the calculation affected? Does the result for the critical current change?

Exercise 2: Superconducting wires: The intermediate state.

In the first exercise sheet we have calculated the critical current I_c of a superconducting wire based on its critical field H_c. For larger currents $I > I_c$, however, the situation is not as straightforward as one might expect.

Consider a cylindrical wire of radius $R \gg \lambda$ carrying a current $I > I_c$ exceeding the critical value.

(a) Because $H(R) > H_c$, there will be a normal surface layer. However, it is not possible to have a remaining superconducting core of radius $R_c < R$, through which all the current flows. Why?

Comment: What is $H(r)$ of the wire in the initial state, and what is $H(r)$ of the core when there is a normal surface layer?

(b) On the other hand, why is it also not possible to have the whole wire in the normal state?

Comment: Plot $H(r)$.

Let us now consider the so-called *intermediate state*, where both normal and superconducting regions coexist, forming an optimized structure. We will look at the most simple model, first proposed by London. While being oversimplified, it nicely illustrates how such an intermediate state may work.

(c) For $R_0 \leq r < R$ the wire is in the normal state. For $r \leq R_0$ there is a core region in an intermediate (mixed) state, where $H(r) = H_c$, and with a fraction r/R_0 of the conduction path parallel to the axis in the normal state while the rest is superconducting, see Fig. 1.

The normal regions, with resistivity ρ, also have to carry some current, or we again have the impossible state described in (a). This requires a longitudinal electric field $\boldsymbol{E} = E\hat{\boldsymbol{z}}$.

The structure is assumed constant in time and we neglect all screening effects.

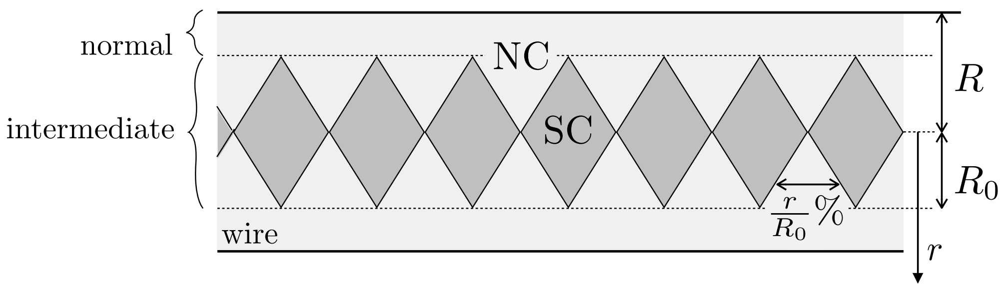

Figure 1: The structure of the intermediate state of a superconducting wire when $I > I_c$. The light grey region labelled NC is normal and the dark grey region labelled SC is superconducting.

We first consider the core region $r \leq R_0$.

(i) What is $I(r)$, the total current inside a radius r? What is the current density $J(r)$?

(ii) The electric field does not depend on r, why? What is $J(r)$ as a function of E and ρ?

(iii) Using the above results, what is the radius R_0 as a function of ρ, H_c and E?

(iv) What fraction $I_{\text{core}} < I$ of the total current can be carried in the core region, again as a function of ρ, H_c and E?

We now turn to the outer, normal layer $R_0 \leq r < R$.

(v) What fraction $I_{\text{outer}} < I$ of the total current is carried in the outer region?

Combining the above results, you have now an expression for the total current $I = I_{\text{core}} + I_{\text{outer}}$.

(vi) What is the electric field E as a function of I?

(vii) Associating the electric field with a resistance R, and where the resistance of a fully normal wire is R_n, write and plot this result as a fraction R/R_n for the two cases $I \lesseqgtr I_c$ as a function of I/I_c.

(viii) Write and plot the fraction R/R_n as a function of T.
Comment: $I_c \propto H_c$*; see Exercise Sheet 1 for* $H_c(T)$*; expansion for* $T \sim T_c$.

Any geometry, except an infinitely long cylinder in a field parallel to its axis, has such an intermediate state. In the next exercise sheet we will see that for some shapes, due to the demagnetization factor*, there are interesting effects already for an external field* $H_{ext} < H_c$ *below the critical value.*