# Quantum Information Theory
# Series 12



**Exercise 12.1 Entropic uncertainty relations**

In the following consider two parties, A and B, that share a state $\rho_{AB}$ and a third party C that has the purification of this in his system C. Hence, pure state $\rho_{ABC}$ describes the shared state between the three people. Use the uncertainty relation for a tripartite pure state $\rho_{ABC}$: $H(X|B)+H(Z|C) \geq -\log c(X,Z)$, with X and Z being orthonormal bases corresponding to different measurements on system A, and c is the maximum overlap between the basis.

a) Show that the overlap $c(X,Z) = 1/2$ between the X and Z Pauli-operator measurements, described by the bases $\{|+\rangle, |-\rangle\}$ and $\{|0\rangle, |1\rangle\}$ respectively.

b) if $\rho_{AB}$ is a maximally entangled two-qubit state $|\psi^+\rangle\langle\psi^+|$ where $|\psi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$, and A performs a X or Z measurement, show that no matter what state C has, C has maximum uncertainty about A's post-measurement state.

c) Think about what happens if A and B do not share a pure state - could then C have some information about A's post measurement state? What if A and B have a pure state, but not maximally entangled one?

**Exercise 12.2 Entanglement and teleportation**

Assume that $|\psi\rangle_S$ is the state A wants to teleport to B. Formally, in teleportation protocol we have three systems $H_S \otimes H_A \otimes H_B$. Here we assume all three are qubits. Initial state is:

$$|\psi\rangle_S \otimes \frac{1}{\sqrt{2}}(|0_A 0_B\rangle + |1_A 1_B\rangle)$$

, hence A and B share fully entangled Bel state, and the total state is separable wrt to S. In the lecture protocol was done for pure states, and here will show that teleportation also works for mixed states $\rho_S$, and also in the case when $\rho_S$ is entangled with some reference system.

a) Look at the state of the system after Alice's measurement but before she communicates her results to Bob. At this point, we know that Bob's state is in one of four possible states that are related to $|\psi\rangle_S$. Show that we still cannot extract any information on $|\psi\rangle_S$ out of Bobs state by calculating it as a probabilistic mixture of the four possible states.

b) Show that the pure states span the space of Hermitian matrices.

c) Show that teleportation also works for mixed states.

d) Now assume that $\rho_S$ is entangled with some reference system R that A and B do not control. Let $\rho = Tr_R|\phi\rangle\langle\phi|_{SR}$. Show that if one applies teleportation protocol on $H_S \otimes H_A \otimes H_B$ gives a final state $|\phi\rangle$ on $H_B \otimes H_R$. This implies that quantum teleportation preserves entanglement. Hence if two parts of an entangled system are transmitted one after the other, the joint system gets teleported accurately.