Phase Transitions and Critical Phenomena

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Exercise Sheet 13

HS 14
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Problem 1 Vertex renormalization of the n-component model

Consider again the n-component model from the lecture with a quartic term

$$u \int d^d \boldsymbol{r} F_{ijk\ell} \phi_i(\boldsymbol{r}) \phi_j(\boldsymbol{r}) \phi_k(\boldsymbol{r}) \phi_\ell(\boldsymbol{r}) \tag{1}$$

where $F_{ijk\ell}$ is the completely symmetric tensor of rank four,

$$F_{ijk\ell} = \frac{1}{3} \left(\delta_{ij}\delta_{k\ell} + \delta_{ik}\delta_{j\ell} + \delta_{i\ell}\delta_{j,k} \right) . \tag{2}$$

Prove the identity

$$\sum_{k\ell} \left(F_{abk\ell}F_{k\ell cd} + F_{ack\ell}F_{k\ell bd} + F_{adk\ell}F_{k\ell bc} \right) = \frac{n+8}{3} F_{abcd} \tag{3}$$

that is needed to find the RG equation for the quartic coefficient u.

Problem 2 ε-expansion of the n-component model

Use identity (3) for the vertex renormalization and identity

$$\sum_k F_{ijkk} = \frac{n+2}{3} \delta_{ij} \tag{4}$$

for the propagator renormalization to arrive at the following RG equations for the n-component model,

$$\frac{du}{d\xi} = -\frac{n+8}{9} u^2 \tag{5}$$

$$\frac{d\tau}{d\xi} = -\frac{n+2}{9} u\tau \tag{6}$$

Use these to derive the critical exponent γ in $4 - \varepsilon$ dimensions to the linear order in ε.