Phase Transitions and Critical Phenomena

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Exercise Sheet 3

HS 14
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Problem 1 Two-component superconductivity in tetragonal crystals

Tetragonal crystals may exhibit superconductivity with a two component order parameter

$$\eta = (\eta_1, \eta_2). \tag{1}$$

The Landau free energy of such a superconducting order parameter is

$$F = \alpha\eta \cdot \eta^* + \frac{\beta_1}{2}(\eta \cdot \eta^*)^2 + \frac{\beta_2}{2}|\eta \cdot \eta|^2 + \frac{\beta_3}{2}\left(|\eta_1|^4 + |\eta_2|^4\right). \tag{2}$$

Find all possible superconducting phases as function of the coefficients $\beta_1, \beta_2, \beta_3$.

Problem 2 Susceptibility near the Néel temperature

Consider the Landau free energy of spins on a bipartite lattice with external field $\boldsymbol{H}$

$$F = \frac{a}{2}\left(M_1^2 + M_2^2\right) + A\boldsymbol{M}_1 \cdot \boldsymbol{M}_2 + \frac{b}{4}\left(M_1^4 + M_2^4\right) - \boldsymbol{H} \cdot \boldsymbol{M} \tag{3}$$

where $\boldsymbol{M}_1, \boldsymbol{M}_2$ are magnetizations of spins on the two sublattices and $\boldsymbol{M} = \boldsymbol{M}_1 + \boldsymbol{M}_2$ is their sum.

We argued in the lecture that close to the Néel temperature T_N

$$A = a - \alpha(T - T_\mathrm{N}) \tag{4}$$

Find the magnetic susceptibilities $\chi_\parallel, \chi_\perp$ of the system near the Néel temperature for a field oriented parallel and perpendicular to the order parameter.