Exercise 1. *D dimensional integrals*

Starting from

$$\int \frac{d^D k}{(2\pi)^D} \frac{1}{(k^2 + 2kq - m^2)^n} = \frac{i(-1)^n}{(4\pi)^{D/2}} \frac{\Gamma(n - \frac{D}{2})}{\Gamma(n)} (q^2 + m^2)^{\frac{D}{2} - n} \tag{1}$$

and differentiating with respect to q^μ and q^ν compute

$$\int \frac{d^D k}{(2\pi)^D} \frac{k^\mu}{(k^2 + 2kq - m^2)^n}$$
$$\int \frac{d^D k}{(2\pi)^D} \frac{k^\mu k^\nu}{(k^2 + 2kq - m^2)^n}$$

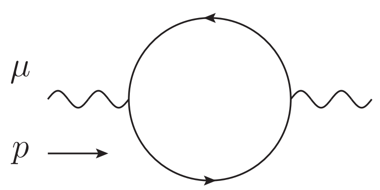
Hence, by setting $q \to 0$ show that

$$\int \frac{d^D k}{(2\pi)^D} \frac{k^\mu k^\nu}{(k^2 - m^2)^n} = \frac{g^{\mu\nu}}{2(n-1)} \int \frac{d^D k}{(2\pi)^D} \frac{1}{(k^2 - m^2)^{n-1}} \tag{2}$$

Verify eq. (2) by contracting with $g_{\mu\nu}$, using $k^2 = (k^2 - m^2) + m^2$ and inserting the explicit result given in eq. (1) with $q^\mu = 0$.

Exercise 2. *Vacuum polarization*

The vacuum polarization $\Pi^{\mu\nu}$ at one loop is given by the single one-particle irreducible diagram



$\equiv i\,\Pi^{\mu\nu}$

with no propagators for the external photon legs. By doing an explicit calculation using dimensional regularization in $D = 4 - 2\epsilon$ dimensions, show that the vacuum polarization can be written as

$$i\,\Pi^{\mu\nu} = i\,(g^{\mu\nu} p^2 - p^\mu p^\nu)\,\Pi(p^2)$$

and that the leading quadratic singularities cancel. Show that upon expansion in ϵ we have

$$\Pi(p^2) = \frac{\bar{e}_0^2}{12\pi^2} \left[-\frac{1}{\epsilon} + \gamma_E - \ln(4\pi) + 6 \int_0^1 d\alpha\, \alpha(1-\alpha) \ln\left(\frac{m_0^2 - \alpha(1-\alpha)p^2}{\mu^2} \right) \right] + \mathcal{O}(\epsilon)$$

where $\bar{e}_0 = e_0 \mu^{-\epsilon}$ is the dimensionless bare coupling.