

marked sections  
not part of exam

# Contents

|  |           |
|--|-----------|
| <b>1 Classical statistical Physics</b>   | <b>6</b>  |
| 1.1 Gibbsian concept of ensembles . . . . .  | 6         |
| 1.1.1 Liouville Theorem . . . . .  | 7         |
| 1.1.2 Equilibrium system . . . . .   | 8         |
| 1.2 Microcanonical ensemble . . . . .  | 8         |
| 1.2.1 Entropy . . . . .  | 9         |
| 1.2.2 Relation to thermodynamics . . . . .   | 10        |
| 1.2.3 Ideal gas - microcanonical treatment . . . . .                               | 11        |
| 1.3 Canonical ensemble . . . . .   | 13        |
| 1.3.1 Thermodynamics . . . . .   | 14        |
| 1.3.2 Equipartition law . . . . .  | 15        |
| 1.3.3 Ideal gas - canonical treatment . . . . .                                    | 15        |
| 1.4 Grand canonical ensemble . . . . .   | 16        |
| 1.4.1 Relation to thermodynamics . . . . .   | 17        |
| 1.4.2 Ideal gas - grand canonical treatment . . . . .                              | 18        |
| 1.4.3 Chemical potential in an external field . . . . .                            | 18        |
| 1.5 Fluctuations . . . . .   | 19        |
| 1.5.1 Energy . . . . .   | 19        |
| 1.5.2 Particle number . . . . .  | 20        |
| 1.5.3 Magnetization . . . . .  | 21        |
| <b>2 Quantum Statistical Physics</b>   | <b>22</b> |
| 2.1 Basis of quantum statistical physics . . . . .                                 | 22        |
| 2.2 Density matrix . . . . .   | 23        |
| 2.3 Ensembles in quantum statistics . . . . .                                      | 24        |
| 2.3.1 Microcanonical ensemble . . . . .  | 24        |
| 2.3.2 Canonical ensemble . . . . .   | 24        |
| 2.3.3 Grand canonical ensemble . . . . .   | 25        |
| 2.4 Ideal quantum paramagnet - canonical ensemble . . . . .                        | 25        |
| 2.4.1 Spin 1/2 . . . . .   | 25        |
| 2.4.2 Spin S - classical limit . . . . .   | 26        |
| 2.5 Ideal quantum gas - grand canonical ensemble . . . . .                         | 28        |
| 2.6 Properties of Fermi gas . . . . .  | 30        |
| 2.6.1 High-temperature and low-density limit . . . . .                             | 31        |
| 2.6.2 Low-temperature and high-density limit: degenerate Fermi gas . . . . .       | 32        |
| 2.6.3 Spin-1/2 Fermions in a magnetic field . . . . .                              | 34        |
| 2.7 Bose gas . . . . .   | 35        |
| 2.7.1 Bosonic atoms . . . . .  | 35        |
| 2.7.2 High-temperature and low-density limit . . . . .                             | 35        |
| 2.7.3 Low-temperature and high-density limit: Bose-Einstein condensation . . . . . | 36        |
| 2.8 Photons and phonons . . . . .  | 39        |
| 2.8.1 Blackbody radiation - photons . . . . .                                      | 41        |

|          |  |            |
|----------|--|------------|
| 2.8.2    | Phonons in a solid . . . . .   | 43         |
| 2.9      | Diatomlic molecules . . . . .  | 44         |
| <b>3</b> | <b>Identical Quantum Particles - Formalism of Second Quantization</b>          | <b>48</b>  |
| 3.1      | Many-body wave functions and particle statistics . . . . .                     | 48         |
| 3.2      | Independent, indistinguishable particles . . . . .                             | 49         |
| 3.3      | Second Quantization Formalism . . . . .  | 50         |
| 3.3.1    | Creation- and annihilation operators . . . . .                                 | 50         |
| 3.3.2    | Field operators . . . . .  | 52         |
| 3.4      | Observables in second quantization . . . . .                                   | 53         |
| 3.5      | Equation of motion . . . . .   | 55         |
| 3.6      | Correlation functions . . . . .  | 56         |
| 3.6.1    | Fermions . . . . .   | 56         |
| 3.6.2    | Bosons . . . . .   | 58         |
| 3.7      | Selected applications . . . . .  | 60         |
| 3.7.1    | Spin susceptibility . . . . .  | 60         |
| 3.7.2    | Bose-Einstein condensate and coherent states . . . . .                         | 61         |
| 3.7.3    | Phonons in an elastic medium . . . . .   | 65         |
| <b>4</b> | <b>One-dimensional systems of interacting degrees of freedom</b>               | <b>70</b>  |
| 4.1      | Classical spin chain . . . . .   | 70         |
| 4.1.1    | Thermodynamics . . . . .   | 70         |
| 4.1.2    | Correlation function . . . . .   | 71         |
| 4.1.3    | Susceptibility . . . . .   | 73         |
| 4.2      | Interacting lattice gas . . . . .  | 74         |
| 4.2.1    | Transfer matrix method . . . . .   | 74         |
| 4.2.2    | Correlation function . . . . .   | 76         |
| 4.3      | Long-range order versus disorder . . . . .                                     | 77         |
| <b>5</b> | <b>Phase transitions</b>   | <b>79</b>  |
| 5.1      | Ehrenfest classification of phase transitions . . . . .                        | 79         |
| 5.2      | Phase transition in the Ising model . . . . .                                  | 81         |
| 5.2.1    | Mean field approximation . . . . .   | 81         |
| 5.2.2    | Instability of the paramagnetic phase . . . . .                                | 82         |
| 5.2.3    | Phase diagram . . . . .  | 85         |
| 5.3      | Gaussian transformation . . . . .  | 86         |
| 5.3.1    | Correlation function and susceptibility . . . . .                              | 89         |
| 5.4      | Ginzburg-Landau theory . . . . .   | 91         |
| 5.4.1    | Ginzburg-Landau theory for the Ising model . . . . .                           | 91         |
| 5.4.2    | Critical exponents . . . . .   | 93         |
| 5.4.3    | Range of validity of the mean field theory - Ginzburg criterion . . . . .      | 94         |
| 5.5      | Self-consistent field approximation . . . . .                                  | 95         |
| 5.5.1    | Renormalization of the critical temperature . . . . .                          | 95         |
| 5.5.2    | Renormalized critical exponents . . . . .                                      | 97         |
| 5.6      | Long-range order - Peierls' argument . . . . .                                 | 98         |
| 5.6.1    | Absence of finite-temperature phase transition in the 1D Ising model . . . . . | 98         |
| 5.6.2    | Long-range order in the 2D Ising model . . . . .                               | 99         |
| <b>6</b> | <b>Superfluidity</b>   | <b>101</b> |
| 6.1      | Quantum liquid Helium . . . . .  | 101        |
| 6.1.1    | Superfluid phase . . . . .   | 102        |
| 6.1.2    | Collective excitations - Bogolyubov theory . . . . .                           | 104        |
| 6.1.3    | Gross-Pitaevskii equations . . . . .   | 108        |

|          |  |            |
|----------|--|------------|
| 6.2      | Berezinskii-Kosterlitz-Thouless transition . . . . .   | 111        |
| 6.2.1    | Correlation function . . . . .   | 111        |
| <b>7</b> | <b>Linear Response Theory</b>  | <b>115</b> |
| 7.1      | Linear Response function . . . . .   | 115        |
| 7.1.1    | Kubo formula - retarded Green's function . . . . .   | 116        |
| 7.1.2    | Information in the response function . . . . .   | 117        |
| 7.1.3    | Analytical properties . . . . .  | 118        |
| 7.1.4    | Fluctuation-Dissipation theorem . . . . .  | 119        |
| 7.2      | Example - Heisenberg ferromagnet . . . . .   | 121        |
| 7.2.1    | Tyablikov decoupling approximation . . . . .   | 122        |
| 7.2.2    | Instability condition . . . . .  | 123        |
| 7.2.3    | Low-temperature properties . . . . .   | 124        |
| <b>8</b> | <b>Renormalization group</b>   | <b>125</b> |
| 8.1      | Basic method - Block spin scheme . . . . .   | 125        |
| 8.2      | One-dimensional Ising model . . . . .  | 127        |
| 8.3      | Two-dimensional Ising model . . . . .  | 129        |
| <b>A</b> | <b>2D Ising model: Monte Carlo method and Metropolis algorithm</b>   | <b>133</b> |
| A.1      | Monte Carlo integration . . . . .  | 133        |
| A.2      | Monte Carlo methods in thermodynamic systems . . . . .   | 133        |
| A.3      | Example: Metropolis algorithm for the two site Ising model . . . . .   | 134        |
| <b>B</b> | <b>High-temperature expansion of the 2D Ising model: Finding the phase transition with Padé approximants</b> | <b>137</b> |
| B.1      | High-temperature expansion . . . . .   | 137        |
| B.2      | Finding the singularity with Padé approximants . . . . .   | 139        |