

## 11 AdS/CFT Correspondence

Conjectured exact duality between string theory and CFT.

- Remarkable!
- Precise formulation of a string/gauge duality.
- Holographic. Different number of spacetime dimensions.
- Main example:  $AdS_5 \times S^5$  string and  $\mathcal{N} = 4$  SYM.

### 11.1 Stack of D3-Branes

Consider 3-brane solution of IIB supergravity (4  $x$  ||, 6  $y$   $\perp$ )

$$ds^2 = h^{-1/2} dx^2 + h^{1/2} dy^2, \quad H_5 = h^{-2} dh dx^4 + h^{-2} *(dh dx^4),$$

with harmonic function  $h(y) = 1 + \alpha N/|y|^4$ .

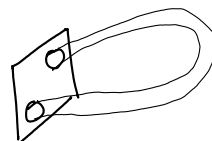
IIB string theory background with stack of  $N$  D3-branes. Low-energy brane physics described by  $U(N)$   $\mathcal{N} = 4$  SYM.

Now approach brane at  $y = 0$ . Alternatively send  $N \rightarrow \infty$ .

- Harmonic function limits to  $h(y) = \alpha N/|y|^4$ .
- Background becomes  $AdS_5 \times S^5$  with 5-form flux.
- $S^5$  at constant  $|y|$ .  $AdS_5$  combined from  $x$  and  $|y|$ .

Claims: AdS/CFT correspondence (Maldacena)

- 3-brane at boundary of  $AdS_5$  space.
- Non-brane modes decouple.
- Boundary physics described exactly by  $U(N)$   $\mathcal{N} = 4$  SYM.
- Open string on boundary can probe bulk  $AdS_5 \times S^5$  strings.



- Precise matching of all observables in both models.
- Map of coupling constants  $(\kappa/R, g_s)$  with  $(g_{YM}, N)$ .

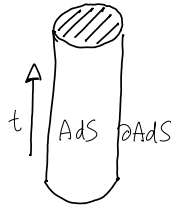
### 11.2 Anti-de Sitter Geometry

Anti-de Sitter space  $AdS_d$  is curved spacetime:

- Constant scalar curvature.
- Analogous to sphere and hyperbolic space

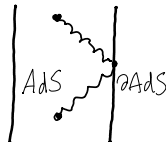
curvature	+	-
Euclidean	S	H
Minkowski	dS	AdS

- Isometry group:  $SO(d-1, 2)$ . Same as conformal group in  $d-1$  dimensions.
- Topology: Solid cylinder  $\mathbb{R} \times D^{d-1}$



Boundary: Cylinder surface  $\mathbb{R} \times S^{d-1}$ .

- time-like geodesics never reach boundary.
- space-like geodesics reach boundary at infinite distance.
- light-like geodesics reach boundary in finite time. bulk and boundary interact via massless fields.



### 11.3 $\mathcal{N} = 4$ Super-Yang-Mills

Maximally supersymmetric gauge theory in 4D. Dimensional reduction from  $\mathcal{N} = 1$  SYM in  $D = 10$ . Fields:

- gauge field,
- 4 adjoint Dirac fermions,
- 6 adjoint scalars.

Remarkable properties:

- no running coupling,  $\beta = 0$ .
- exact 4D superconformal symmetry; 4D (S)CFT.
- ...

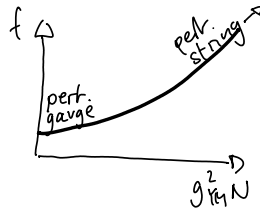
### 11.4 Tests

Want to test AdS/CFT correspondence. Predictions:

- String spectrum matches with spectrum of local operators.
- String and gauge correlation functions match.

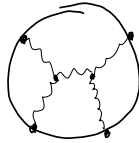
Problem: Strong/weak coupling duality.

- Weakly coupled strings is strongly coupled gauge theory.
- Weakly coupled gauge theory is strongly coupled strings.



Test BPS quantities, protected (independent of coupling).

- Supergravity modes agree with BPS operators.
- Supergravity correlators match with BPS correlators.



What about other quantities?

- String and gauge theory appear integrable at large  $N$ .
- Integrability: Hidden symmetry to constrain dynamics.
- Can compute observables efficiently even at finite coupling.
- Precise agreement found in all performed tests.

Other tests performed, e.g. Wilson loops vs. string area.