6 Open Strings and D-Branes

So far we have discussed closed strings. The alternative choice is open boundary conditions.

6.1 Neumann Boundary Conditions

Conventionally $0 \leq \sigma \leq \pi$ and we have to discuss $\sigma = 0, \pi$. Start again in conformal gauge

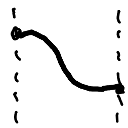
$$S = \frac{1}{2\pi\kappa^2} \int d^2\xi \, \tfrac{1}{2}\eta^{ab} \, \partial_a X \cdot \partial_b X.$$

Variation including boundary terms due to partial integration

$$\begin{aligned}
\delta S &= \frac{1}{2\pi\kappa^2} \int d^2\xi \, \eta^{ab} \, \partial_a \delta X \cdot \partial_b X \\
&= \frac{1}{2\pi\kappa^2} \int d^2\xi \, \partial_a(\eta^{ab} \, \delta X \cdot \partial_b X) - \ldots \\
&= \frac{1}{2\pi\kappa^2} \int d\tau \big(\delta X(\pi) \cdot X'(\pi) - \delta X(0) \cdot X'(0)\big) - \ldots .
\end{aligned}$$

Boundary e.o.m. imply Neumann conditions (alternative later)

$$X'(0) = X'(\pi) = 0.$$



Virasoro constraints

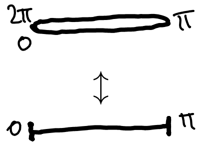
$$X' \cdot \dot{X} = X'^2 + \dot{X}^2 = 0$$

imply that end points move at speed of light $\dot{X}^2 = 0$. (no free ends of analogous soap film: $\dot{X}^2 = 0$ implies $\dot{X} = 0$.)

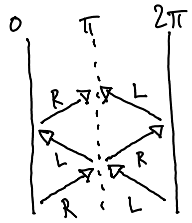
6.2 Solutions and Spectrum

Same equations in the string bulk, recycle solution.

Doubling Trick. Map two copies of open string to closed string twice as long: $\sigma \equiv 2\pi - \sigma$. Gluing condition $X' = 0$ at $\sigma = 0, \pi$ implies



$$\alpha_n^{\mathrm{L}} = \alpha_n^{\mathrm{R}}.$$



Left movers are reflected into right movers at boundary. One copy of oscillators and Virasoro algebra

$$X^\mu = x^\mu + 2\kappa^2 p^\mu \tau + \sum_{n\neq 0} \frac{i\kappa}{\sqrt{2}n}\, \alpha_n^\mu \left(\exp(-in\xi^{\mathrm{L}}) + \exp(-in\xi^{\mathrm{R}})\right).$$

(momentum p is doubled because σ integration is halved)

Quantisation. Analogous to closed strings. Same anomaly conditions $D = 26$, $a = 1$ (from bulk). Resulting spectrum (note different prefactor due to p).

$$M^2 = \frac{1}{\kappa^2}\,(N - a).$$

Only single copy of oscillators at each level.

- level 0: singlet tachyon (of half "mass").
- level 1: massless vector: Maxwell field.
- level 2: massive spin-2 field $\square\!\square$.
- ...

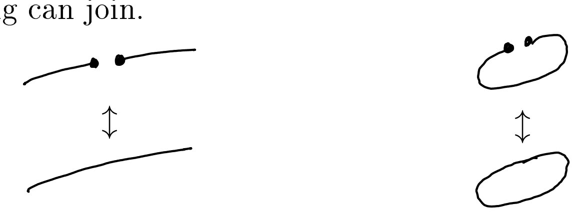
Same as discussion for closed string without squaring!

Massless modes are associated to local symmetries:

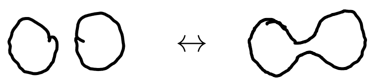
- of open string are spin-1 gauge fields,
- of closed string are spin-2 gravitation fields.

String Interactions. Open and closed strings interact:

- Two ends of string can join.



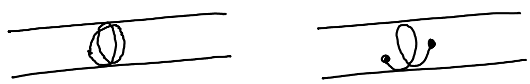
  Open strings must include closed strings. Different "vacuum" states $|0; q\rangle_{\mathrm{c}}$ and $|0; q\rangle_{\mathrm{o}}$ in same theory.
- Opening of string can be suppressed. Closed string can live on their own.



String theory always contains gravity; May or may not include gauge field(s).

6.3 Dirichlet Boundary Conditions

Now consider compactification for open strings. Almost the same as for closed string. No winding modes because open string can unwind.



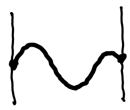
T-Duality. What about applying T-duality? Introduce dual fields

$$X' = \dot{\tilde{X}}, \qquad \dot{X} = \tilde{X}'.$$

Boundary conditions translate to

$$X'^{\mu} = 0 \quad (\mu = 0, \dots, 24), \qquad \dot{\tilde{X}}^{25} = 0.$$

Dirichlet boundary condition for dual coordinate $\tilde{X}^{25}$. Corresponds to alternate choice of boundary e.o.m. $\delta X^{25} = 0$.



KK modes turn into winding modes:

$$\Delta \tilde{X}^{25} = \int d\sigma \tilde{X}'^{25} = \int d\sigma \dot{X}^{25} = 2\pi\kappa^2 p_{25} = \frac{2\pi\kappa^2 n}{R} = 2\pi n \tilde{R}.$$

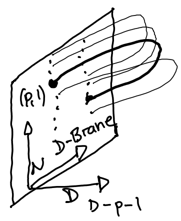
Strings start and end at same $x_0 \equiv x_0 + 2\pi\tilde{R}$. Note: No momentum $\tilde{P}_{25}$ because position fixed. Role of KK and winding exchanged.

Dirichlet condition modifies oscillator relation:

$$\alpha_n^{\mathrm{L},25} = -\alpha_n^{\mathrm{R},25}.$$

Although Dirichlet condition $\tilde{X}^{25} = \text{const.}$ appears unnatural, it has to be part of string theory (on compact spaces).

D-Branes. Take seriously.



At boundary can choose:
- Neumann condition $X'^{\mu} = 0$ or
- Dirichlet condition $X^{\mu} = \text{fixed}$

for each direction μ individually.

Geometrical picture: String ends confined to Dp-branes.

- $p+1$ dimensional $(p, 1)$ submanifolds of spacetime.
- Dirichlet conditions for $D - p - 1$ orthogonal directions.
- Neumann conditions for $p + 1$ parallel directions.
- D-branes can be curved (normal depends on position).

T-duality maps between Dp and D$(p \pm 1)$ branes.

Pure Neumann conditions are spacetime-filling D-brane.

Strings propagate on backgrounds with D-branes:

- spacetime bulk curvature governs string bulk propagation,
- D-branes govern string end propagation.

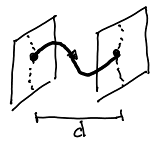
Even more: Will continue discussion later.

6.4 Multiple Branes

Can have multiple branes of diverse types. Open strings stretch between two branes.

Parallel Branes. Simplest case: Two parallel planar Dp-branes located at $X^{25} = 0, d$ in non-compact Minkowski space

$$X^\mu = 2\kappa^2 p^\mu \tau + \text{modes}, \quad X^{25} = \frac{\sigma d}{\pi} + \text{modes}.$$



Resulting (quantum) mass spectrum in $p+1$ dimensions

$$M^2 = \frac{d^2}{4\pi^2\kappa^4} + \frac{1}{\kappa^2}(N - a).$$

- Spin-1 particle at level-1 with mass $M = d/2\pi\kappa^2$.
- Vector massless at coincident branes.
- Tachyon for $d < 2\pi\kappa$: Instability for nearby D-branes.

Multiple Branes. Consider now N parallel branes.

There are N^2 types of open string (and 1 closed): String vacua distinguished by Chan–Paton factors

$$|0; q; a\bar{b}\rangle_{\text{o}}, \qquad a, \bar{b} = 1, \ldots, N.$$

with general mass formula

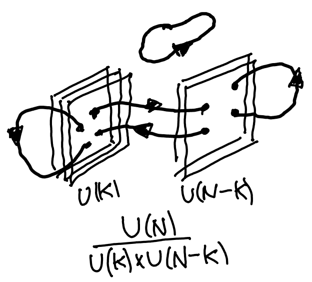
$$M^2_{a\bar{b}} = \frac{d^2_{a\bar{b}}}{4\pi^2\kappa^4} + \frac{1}{\kappa^2}(N - a).$$

Consider vector particles at level 1 with mass $d_{a\bar{b}}/2\pi\kappa^2$.

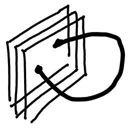
- Always N massless vectors. Gauge symmetry: $U(1)^N$.
- K coincident branes contribute K^2 massless vectors. Enhanced gauge symmetry $U(1)^K \to U(K)$.
- Massive vectors indicate spontaneously broken symmetries.

Geometric picture of gauge symmetries:

- Stack of N branes have local $U(N)$ symmetry.
- Separating branes breaks symmetry to $U(K) \times U(N-K)$.
- Creates $2K(N-K)$ massive vectors.



Can also produce $SO(N)$ and $Sp(N)$ symmetries: Unoriented strings, strings on orientifolds (spacetime involution paired with orientation reversal).



Brane Worlds. Can design many different situations.

Combine:

- non-compact dimensions,
- D-branes,
- intersections of D-branes and non-compact dimensions,
- orientifold action.

Consider physics:

- along non-compact dimensions,
- within D-branes.

Qualitative features:

- Massless vectors indicate gauge symmetries.
- Light vectors indicate spontaneous symmetry breaking.
- Tachyons indicate instabilities of D-branes or spacetime.

String theory becomes framework analogous to QFT:

- D-brane arrangements and compact directions (discrete),
- moduli for D-branes and non-compact spaces (continuous).

Physics: Try to design the standard model at low energies.

Mathematics: Dualities relate various situations.