5 Compactification

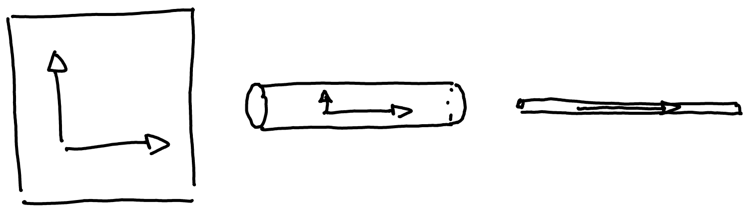
We have seen that the closed string spectrum contains:

- 1 tachyonic scalar particle (wrong vacuum).
- massless gravitons and few other particles.
- tower of particles of increasing mass (inaccessible).

But: $D = 26$ dimensions, way too many! Gauss law: gravitational force: $F \sim 1/A \sim 1/r^{24}$ not $1/r^2$.

5.1 Kaluza–Klein Modes

Idea: Compactify 22 dimensions to microscopic size. Large distances only for 4 remaining dimensions. Small compact dimensions almost unobservable.



Compactify one dimension to a circle of radius R

$$X^{25} \equiv X^{25} + 2\pi R.$$

Quantum mechanical momentum quantised

$$P_{25} = \frac{n}{R}\,.$$

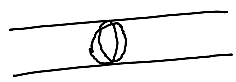
Effectively tower of massive particles $M_{25}^2 = M_{26}^2 + n^2/R^2$:

- Zero mode $n = 0$ has original mass. Massless mode observable.
- Higher modes are massive, $M \simeq 1/R$. For very small R: practically unobservable.

Low-energy physics can be effectively four-dimensional.

5.2 Winding Modes

Peculiarity of strings on compact spaces: Winding.



Consider again one compact direction $X^{25} =: X \equiv X + 2\pi R$. Need to relax periodicity: $X(\sigma + 2\pi) = X(\sigma) + 2\pi R m$.

$$X_{\mathrm{L/R}} = \tfrac{1}{2}x + \tfrac{1}{2}\kappa^2 \left(\frac{n}{R} \mp \frac{mR}{\kappa^2}\right) \xi^{\mathrm{L/R}} + \text{modes}.$$

Mass (for propagation in 25 non-compact dimensions)

$$M^2 = \frac{4}{\kappa^2}(N^{\mathrm{L/R}} - a) + \left(\frac{n}{R} \mp \frac{mR}{\kappa^2}\right)^2.$$

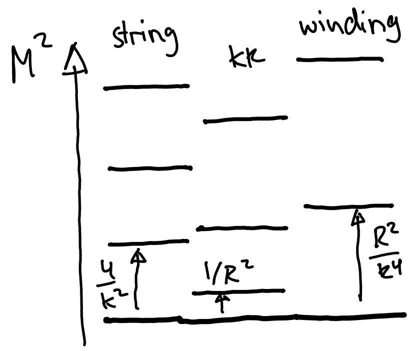
Level matching condition modified by winding

$$N^{\mathrm{L}} - N^{\mathrm{R}} = nm.$$

L/R average formula for mass

$$M^2 = \frac{2}{\kappa^2}(N^{\mathrm{L}} + N^{\mathrm{R}} - 2a) + \frac{n^2}{R^2} + \frac{m^2 R^2}{\kappa^4}.$$

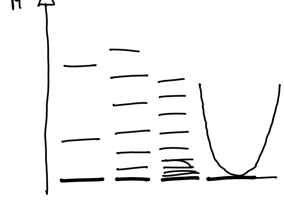
Winding also contributes mass. To hide infinitely many modes: κ, R and κ^2/R small!



"Decompactify" circle as $R \to \infty$:

- Winding modes become very heavy.
- KK modes form become light and continuum.

Note: Also modes with $N^{\mathrm{L}} \neq N^{\mathrm{R}}$ exist (new representations). Additional modes become infinitely heavy at $R \to \infty$.



Can also try to compactify circle $R \to 0$.

- KK modes become very heavy.
- Winding modes become light and form continuum.

Same as for $R \to \infty$ with role of m and n interchanged. Observe: spectrum the same for R and κ^2/R.

Additional dimension remains observable at $R \to 0$! Different from regular point particle with KK only.

5.3 T-Duality

Duality between small and large compactification radius. Can show at Lagrangian level: **T-duality**.

Start with action of 25-direction in conformal gauge

$$\frac{1}{2\pi\kappa^2}\int d^2\xi\, \tfrac{1}{2}\eta^{ab}\,\partial_a X\,\partial_b X$$

Action has global shift symmetry $X \to X + \epsilon$. For winding we would need local shift, let us make the symmetry local ("gauge"), $A_a \to A_a - \partial_a\epsilon$

$$\frac{1}{2\pi\kappa^2}\int d^2\xi \left(\tfrac{1}{2}\eta^{ab}(\partial_a X + A_a)(\partial_b X + A_b) - \varepsilon^{ab}\tilde{X}\partial_a A_b\right).$$

Added two d.o.f. in A_a and one local redundancy. Remove further d.o.f. by demanding $F_{ab} = \partial_a A_b - \partial_b A_a = 0$. through Lagrange multiplier $\tilde{X}$. Done nothing (e.g. $A_a = 0$).

Field A_a is algebraic, integrate out: E.o.m.

$$A_a = -\partial_a X + \eta_{ac}\varepsilon^{cb}\partial_b\tilde{X}$$

Substitute and obtain (up to boundary term)

$$\frac{1}{2\pi\kappa^2}\int d^2\xi \tfrac{1}{2}\eta^{ab}\partial_a\tilde{X}\,\partial_b\tilde{X}.$$

Same as before, but with $\tilde{X}$ instead of X.

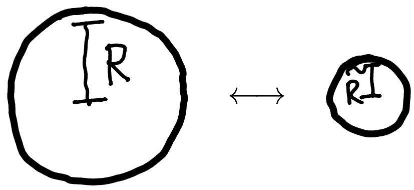
Now can set $A_a = 0$ and obtain the duality relation

$$\partial_a X = \eta_{ac}\varepsilon^{cb}\partial_b\tilde{X}, \quad \text{i.e.} \quad \dot{X} = \tilde{X}', \quad X' = \dot{\tilde{X}}.$$

For the standard solution X we find the dual $\tilde{X}$

$$X = x + \kappa^2\frac{n}{R}\tau + mR\sigma + \text{modes}.$$
$$\tilde{X} = \tilde{x} + mR\tau + \kappa^2\frac{n}{R}\sigma + \text{modes}.$$

Duality interchanges $R \leftrightarrow \tilde{R} = \kappa^2/R$ and $m \leftrightarrow n$.



Effectively $R = \kappa$ is minimum compactification radius. It is indeed a special "self-dual" point. Duality between two models turns into enhanced symmetry.

$R = \kappa$ is minimum length scale in string theory: quantisation of spacetime in quantum gravity.

5.4 General Compactifications

So far compactified one dimension: Only circle or interval. Many choices and parameters for higher compactifications.

- sphere S^n,
- product of spheres $S^a \times S^{n-a}$, different radii,
- torus T^n, $3n - 3$ moduli (radii, tilts),
- other compact manifolds.

Low-lying modes determined by manifold (bell).

- Compactification determines observable spectrum.
- Goal: find correct manifold to describe SM.
- Massless modes correspond to gauge symmetries.
- Superstrings: CY 3-fold preserved 1 susy.