Statistical Physics
Exercise 6

Exercise 6.1 The Ising Paramagnet

Consider N localized non-interacting magnetic moments which take the values $s_i = \pm s$. In the presence of an external magnetic field h the Hamiltonian of this system is given by

$$\mathcal{H} = -\sum_i h s_i .$$

Calculate the free energy $F(T,h)$, the caloric and thermal equations of state, the heat capacity $C(T,h)$ and the magnetic susceptibility $\chi(T,h)$. Find a relation between the fluctutions in the magnetization ($\Delta m^2 = \langle s_i^2 \rangle - \langle s_i \rangle^2$) and the susceptibility.

Exercise 6.2 Non-interacting Particles in the Gravitational Field

Consider a gas of non-interacting particles at fixed temperature T in the gravitational field

$$V_{\text{grav}}(x, y, z \geq 0) = gz$$

with the *gravitational constant* $g > 0$. The volume of the gas is confined to a vertical, cylindrical vessel (radius R) of infinite height.

a) Using the canonical ensemble, find the Helmholtz free energy, the entropy, the heat capacity, and the internal energy of this system.

b) Interpret the result of the heat capacity via the equipartition law.

c) Consider the system from the viewpoint of local thermal equilibrium. Determine the local (one-particle) density $n(z)$ and the local pressure $p(z)$ and show that the equation of states $p(z) = k_{\text{B}} T n(z)$ holds. Find the expression of the local internal energy density $u(z)$. The local version of the relation (1.17) in the lecture notes takes the form

$$c_p = \left(\frac{\partial u}{\partial T}\right)_n + \left\{ \left(\frac{\partial U}{\partial V}\right)_T + p \right\} \alpha = \left(\frac{\partial u}{\partial T}\right)_n + T \left(\frac{\partial p}{\partial T}\right)_n \alpha \, . \tag{1}$$

Show that thermal expansion coefficient α is given by $1/T$, independent of z. Note that in equation (1) we have used the fact that keeping the specific volume constant is equivalent to a constant local density. Calculate c_p and compare it with the result from part a).

d) Calculate the heat capacity via the variance of $\mathcal{H}$ and interpret the resulting terms.

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