Exercise 1) Entanglement Dilution

Let $|\psi\rangle_{AB}$ be a pure bipartite quantum state with local eigenvalues $(r, 1-r)$, $n \geq 1$, and $\frac{1}{4} \geq \epsilon > 0$.

In the protocol for entanglement dilution Alice locally creates $|\psi\rangle_{AB}^{\otimes n}$ and applies the Schur transform to the A and B part separately. Then she measures the projector

$$P_\epsilon = \sum_{k \in n[1-2r-2\epsilon, 1-2r+2\epsilon]} |k\rangle\langle k| \ ,$$

and its complement.

a) Show that for $n \to \infty$ the probability for Alice to measure P_ϵ goes to one.

b) Estimate how many path ebits there are if Alice measures P_ϵ. For this, first show

$$|h(x) - h(x+\epsilon)| \leq -2\epsilon \log \epsilon \ ,$$

where $h(x)$ denotes the binary entropy function.

Hint: Show that $f(t) = -t \log t$ is concave and from that $|f(t) - f(t+s)| \leq f(s)$ for $0 \leq t \leq 1-s$ and $s \leq \frac{1}{2}$.

c) The next step in the protocol is to exchange the path ebits against ebits shared with Bob and then to teleport all the remaining outputs from the Schur transform on the B systems to Bob. Finally Alice and Bob both apply the inverse Schur transform.

Estimate how many ebits the protocol consumes and how much classical communication is needed.

Exercise 2) Schmidt Coefficients

a) How many non-zero Schmidt coefficients do n ebits have?

b) Show that the number of non-zero Schmidt coefficients can not be increased by local measurements.

Hint: The Schmidt coefficients are just the square roots of the eigenvalues of the local density matrix.