Matter parity is a multiplicatively conserved quantum number defined as

$$P_M = (-1)^{3(B-L)}$$

for each particle in the theory. It is easy to check that the quark and lepton supermultiplets have $P_M = -1$, while the Higgs supermultiplets have $P_M = +1$. Gauge fields and gauginos have $B = L = 0$, so they have $P_M = +1$. A term is allowed in the Lagrangian only if it is even under matter parity. It is easy to see that all the terms in $W_{\Delta L = 1}$ and $W_{\Delta B = 1}$ are forbidden, while all the terms we previously included in the superpotential are allowed.

<u>NOTE</u>. Even if matter parity is an exact symmetry, baryon and lepton number conservation could be violated in the MSSM. However the MSSM does not have renormalizable interactions that violate B or L, if matter parity conservation is assumed.

It is often useful to recast matter parity in terms of R-parity, defined as

$$P_R = (-1)^{3(B-L)+2s},$$

where s is the spin of the particle. Matter parity and R-parity are equivalent, since the product of $(-1)^{2s}$ for the particles involved in an interaction vertex in a theory that conserves angular momentum is always equal to $+1$ (equivalently one can notice that all the terms in the Lagrangian have an even number of fermions).

Particles in the same multiplet do not have the same R-parity, so this symmetry does not commute with supersymmetry, it is an R-symmetry.

R-parity transforms the particles as

$$(\text{SM particle}) \to (\text{SM particle}),$$
$$(\text{superpartner}) \to -(\text{superpartner}).$$

R-parity conservation has a series of extremely important phenomenological consequences:

- The lightest sparticle with $P_R = -1$, called the "lightest supersymmetric particle" or LSP, must be absolutely stable. If the LSP is electrically neutral, it interacts only weakly with ordinary matter and could be a dark matter candidate.
- Each sparticle other than the LSP must eventually decay into a state that contains an odd number of LSP's.
- In collider experiments, sparticles can only be produced in even numbers (usually in pairs).

<u>NOTE</u>. R-parity or matter parity could originate from a gauged U(1) symmetry which is spontaneously broken at high energy to a discrete subgroup.

Where is supersymmetry broken?

Obviously supersymmetry is not present in the low-energy particle spectrum which has been experimentally explored. None of the superpartners of the SM fields, which are included in the MSSM, has been seen, and this implies that supersymmetry, if present in Nature, must be necessarily broken at low-energy.

The simplest possibility to make the MSSM compatible with the experiment would be to have spontaneous susy breaking arising at tree level. However this possibility can be definitely ruled out. There are several considerations which allow us to exclude all possible direct susy-breaking mechanisms in the MSSM.

- A first, simple argument is based on the tree-level supertrace sum rule:

$$\mathrm{Str}\, \mathcal{M}^2 = -2g \langle D^a \rangle \mathrm{Tr}\, T^a .$$

In the MSSM a VEV for D^a (in particular for the $U(1)_Y$ component) is not phenomenologically viable (for example if we include a FI term we would get a VEV for the squarks or sleptons, which do not have a superpotential mass term, and this would lead to color or electric charge breaking), so we can take

$$\underline{\mathrm{Str}\, \mathcal{M}^2 = 0} .$$

This is valid for each set of fields with given color representation and electric charge. In the color triplet sector with electric charge $-e/3$, the only known fermions are the d, s and b quarks, for which

$$m_d^2 + m_s^2 + m_b^2 \simeq (5\,\mathrm{GeV})^2 .$$

According to the sum rule, if there are no other fermions with this color and charge, then the sum of all squared masses for bosons with the same color and charge must equal about $2\cdot(5\,\mathrm{GeV})^2$. This implies that each of the squarks with this color and charge must have a mass not greater than $\sim 7\,\mathrm{GeV}$. The existence of such states is definitely ruled out experimentally.

This argument, however, could be invalidated if there were a fourth generation of quarks.

- A much stronger argument can be obtained by more carefully studying the possible susy breaking mechanisms in the MSSM.

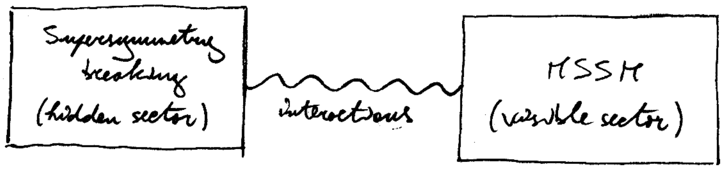
The unbroken conservation of electric charge and color tells us that the only non-zero D-terms are the ones for the $U(1)_Y$ generator (and at most also the t_3 generator of $SU(2)_L$, which is allowed when $SU(2)_L \times U(1)_Y$ is broken to $U(1)_{EM}$). Moreover all the F-terms must vanish for the quark fields (given that they are color triplets) and the VEV's for the squarks must vanish as well.

One can prove that, depending on the sign of the D-terms, there must be either a squark of charge $2e/3$ lighter than the u quark, or a squark of charge $-e/3$ lighter than the d quark. Both this possibilities are experimentally ruled out.

So we are forced to reject the possibility of having susy spontaneously broken at tree level in the MSSM.

13.12.

The above results are not the only arguments which show that susy breaking can not easily happen in the MSSM. The obvious consequence is that we need to assume that susy is broken by some unknown mechanism in an "hidden" sector, which has only small direct coupling with the "visible" sector given by the MSSM.



The breaking of susy in the hidden sector is communicated to the visible sector through some shared interactions, which generate only "soft" susy breaking terms in the visible sector. The assumption that the generated terms are "soft" (that is superrenormalizable) comes from the fact that we want to preserve the protection mechanisms of the MSSM, which avoid the presence of quadratic divergences in the masses coming from loop corrections. We will better discuss this aspect later on.

This mechanism of susy breaking through an hidden sector has some nice advantages. One of these is the fact that, if the mediating interactions are flavor-blind, then the soft terms appearing in the MSSM will not introduce large flavor-changing effects which are not seen experimentally.

Moreover we could also think to a mechanism which explains the difference of scale of the susy breaking scale and for example the Planck scale or a possible unification scale M_X. This can be obtained if susy is broken by non-perturbative effects. In particular, if there is a gauge field with an asymptotically free gauge coupling $g(\mu)$ at the renormalization scale μ and if $g^2(\mu)/8\pi^2 \ll 1$ for $\mu \simeq M_X$, then, by the running, this gauge interaction will become strong at an energy of order

$$M_S \simeq M_X \exp\left(-8\pi^2 b / g^2(M_X)\right),$$

where b is a number of order unity. If susy is broken by this coupling which becomes strong, then a big difference between M_S and M_X can be naturally explained.

For the mechanism which can mediate susy breaking there are two leading candidates. One is provided by the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge interactions themselves, the other is gravitation. These two mechanisms give rather different phenomenological predictions for the scale of susy breaking and for the masses of some superpartners. In the following we will summarize some general results largely independent of the exact implementation of the susy breaking mechanism.

· In <u>gauge-mediated supersymmetry breaking</u> the mediating interactions are the usual electroweak and QCD gauge interactions. The MSSM soft terms come from loop diagrams involving some <u>messenger</u> particles. The messengers are new chiral supermultiplets that couple to a susy-breaking VEV $\langle F \rangle$, and also have $SU(3)_c \times SU(2)_L \times U(1)_Y$ interactions, which provide the necessary connection to the MSSM. Using a rough estimate we get for the induced soft terms

$$m_{soft} \sim \frac{\alpha_a}{4\pi} \frac{\langle F \rangle}{M_{mess}}$$

where $\alpha_a/4\pi = g_a^2/16\pi^2$ is a loop factor for Feynman diagrams involving gauge interactions and M_{mess} is a characteristic scale of the masses of the messenger fields. If $\sqrt{\langle F \rangle}$ and M_{mess} are roughly comparable, then, in order to have $m_{soft} \sim 1\,\mathrm{TeV}$, we need a susy breaking scale

$$\underline{\sqrt{\langle F \rangle} \sim 100\ \mathrm{TeV}.} \qquad \text{(gauge mediated)}$$

· In <u>Planck-scale mediated supersymmetry breaking</u> or <u>gravity-mediated supersymmetry breaking</u> the mediating interactions are the gravitational interactions. The estimate of the susy breaking scale varies according to the susy breaking mechanism.

· If susy is broken in the hidden sector by a VEV $\langle F \rangle$, then the soft terms in the visible sector should be roughly

$$m_{soft} \sim \frac{\langle F \rangle}{M_{Pl}}$$

by dimensional analysis. This is because we know that m_{soft} must vanish in the limit $\langle F \rangle \to 0$ where susy is unbroken, and also in the limit $M_{Pl} \to \infty$ in which gravity becomes irrelevant. This would give

$$\underline{\langle F \rangle \sim 10^{11}\ \mathrm{GeV}.} \qquad \text{(gravity mediated I)}$$

· Another possibility is that the susy breaking order parameter is a <u>gaugino condensate</u> $\langle 0 | \lambda^a \lambda^b | 0 \rangle = \delta^{ab} \Lambda^3 \neq 0$. If the composite field $\lambda^a \lambda^b$ is part of an auxiliary field F for some (possibly composite) chiral superfields, then by dimensional analysis we expect

$$m_{soft} \sim \frac{\Lambda^3}{M_{Pl}^2},$$

with, effectively, $\langle F \rangle \sim \Lambda^3/M_{Pl}$. In this case, the scale of susy breaking should be roughly

$$\underline{\Lambda \sim 10^{13}\ \mathrm{GeV}.} \qquad \text{(gravity mediated II)}$$

· The large difference in the estimates of the susy breaking scale M_S for gauge and gravitational mediation makes an important difference in particle phenomenology and cosmology. Supersymmetry dictates that the graviton must have a partner of spin $3/2$, the gravitino. When susy is broken the gravitino acquires a mass of order M_S^2/M_{Pl} (and it "eats" the Goldstino in order to acquire the additional spin $1/2$ components needed to form a massive spin $3/2$ field).

For gauge-mediated susy breaking the gravitino mass is very small

$$m_{\tilde{g}} \sim \frac{M_S^2}{M_{Pl}} \approx 1\,eV ,$$

so the gravitino would be by far the lightest of the new particles required by supersymmetry. Conservation of R-parity would also imply that this particle is stable.

On the other hand, for gravity-mediated susy breaking the gravitino mass is just of the same order of magnitude as the mass splitting between the known particles and their superpartners, so they would have roughly the same mass as the squarks, sleptons and gauginos. In this case the gravitino may or may not be the lightest supersymmetric partner.

- If the gravitino is the lightest susy particle it could play the role of dark matter. From this observation one can find some limits on the susy breaking scale.

  In the case of gauge-mediation we have a very light gravitino ($m_{\tilde{g}} \sim 1\,eV$) and the bounds require a dark matter candidate to be lighter than $\sim 1\,keV$. Thus this limit is well satisfied, but at the same time the gravitino is too light to give a appreciable contribution to the mass density of the universe.

  In the case of gravity-mediation, the experimental constraints require the gravitino to be heavy enough, roughly $m_{\tilde{g}} > 10\,TeV$. This implies that the susy breaking scale should be $M_S > 10^{11}\,GeV$ in the case of breaking through a vev $\langle F \rangle$, while it should be $M_S > 10^{13}\,GeV$ in the case of breaking through a gaugino condensate.

- <u>Soft supersymmetry breaking interactions</u>

  Let's now discuss what kind of susy breaking terms can be introduced in a susy theory if we want to preserve the cancellation of quadratic divergences in the loop corrections to the masses of the scalars.

  The possible soft susy-breaking terms in a general theory are

$$\mathcal{L}_{soft} = -\left(\frac{1}{2} M_a \lambda^a \lambda^a + \frac{1}{6} a^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} b^{ij} \phi_i \phi_j + t^i \phi_i\right) + h.c. - (m^2)^i_j\, \phi^{j*} \phi_i ,$$

$$\mathcal{L}_{maybe\,soft} = -\frac{1}{2} c_i^{jk} \phi^{*i} \phi_j \phi_k + h.c.$$

They consist of

- gaugino masses M_a for each gauge group,
- scalar squared-mass terms $(m^2)_i^j$ and b^{ij},
- $(\text{scalar})^3$ couplings a^{ijk} and c_i^{jk},
- "tadpole" couplings t^i (this can occur only if ϕ_i is a gauge singlet, so it is absent in the MSSM).

NOTE. We could also include soft mass terms for the chiral supermultiplets fermions, like $\mathcal{L} = -\frac{1}{2} m^{ij} \psi_i \psi_j + h.c.$. However, including such terms would be redundant; they can always be absorbed into a redefinition of the superpotential and the terms $(m^2)_i^j$ and c_i^{jk}.

It has been shown that a softly broken susy theory with $\mathcal{L}_{soft}$ is free of quadratic divergences in quantum corrections to scalar masses to all orders in perturbation theory. The situation is more subtle if one tries to include the terms in $\mathcal{L}_{maybe\ soft}$. If any of the chiral supermultiplets in the theory are singlets under all gauge symmetries, then non-zero c_i^{jk} terms can lead to quadratic divergences, despite the fact that they are superrenormalizable and, therefore, formally soft. This constraint does not apply to the MSSM, which does not have gauge-singlet chiral superfields. Nevertheless, the possibility of c_i^{jk} terms is nearly always neglected, because it is difficult to construct models of spontaneous susy breaking in which the c_i^{jk} are not negligibly small.

NOTE. In the special case of a theory that has chiral supermultiplets that are singlets or in the adjoint representation of a simple factor of the gauge group, there are also possible soft susy-breaking mass terms between the corresponding fermions ψ_a and the gauginos

$$\mathcal{L} = -M_b^a \lambda^a \psi_a + h.c.$$

This is not relevant for the MSSM, which does not have chiral multiplets in the adjoint representation.

A few comments.

- The gaugino masses M_a are always allowed by gauge symmetry.
- The $(m^2)_i^j$ terms are allowed for i, j such that ϕ_i, ϕ^{j*} transform in complex conjugate representations of each other under all gauge symmetries. In particular this is true if $i=j$, so every scalar is eligible to get a mass in this way if susy is broken.
- The a^{ijk}, b^{ij} and t^i terms have the same form of some terms in the superpotential, so they are allowed by gauge invariance if and only if a corresponding superpotential term is allowed.

• Soft supersymmetry breaking in the MSSM.

We have discussed the form of the most general set of soft supersymmetry breaking terms. Now we want to understand what happens in the MSSM. Applying the general recipe to the MSSM we get

$$\mathcal{L}_{SOFT}^{MSSM} = -\frac{1}{2}\left(M_3 \tilde{g}\tilde{g} + M_2 \tilde{W}\tilde{W} + M_1 \tilde{B}\tilde{B} + h.c.\right)$$
$$-\left(\tilde{\bar{u}}\, a_u \tilde{Q} H_u - \tilde{\bar{d}}\, a_d \tilde{Q} H_d - \tilde{\bar{e}}\, a_e \tilde{L} H_d + h.c.\right)$$
$$-\tilde{Q}^+ m_Q^2 \tilde{Q} - \tilde{L}^+ m_L^2 \tilde{L} - \tilde{\bar{u}}\, m_{\bar{u}}^2\, \tilde{\bar{u}}^+ - \tilde{\bar{d}}\, m_{\bar{d}}^2\, \tilde{\bar{d}}^+ - \tilde{\bar{e}}\, m_{\bar{e}}^2\, \tilde{\bar{e}}^+$$
$$-m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - \left(b H_u H_d + h.c.\right).$$

In the above expression

- M_3, M_2 and M_1 are the gluino, wino and bino mass terms.
- The second line contains the $(\text{scalar})^3$ couplings; each of a_u, a_d, a_e is a complex 3x3 matrix in family space, with dimension of [mass], and they are in one-to-one correspondence with the Yukawa couplings of the superpotential.
- The third line consists of squark and slepton mass terms of the $(m^2)_i^j$ type. Each of m_Q^2, $m_{\bar{u}}^2$, $m_{\bar{d}}^2$, m_L^2, $m_{\bar{e}}^2$ is a 3x3 matrix in family space that can have complex entries, but they must be Hermitian, so that the Lagrangian is real.
- In the last line we have supersymmetry-breaking contributions to the Higgs potential; $m_{H_u}^2$ and $m_{H_d}^2$ are squared-mass terms of the $(m^2)_i^j$ type, while b is the only squared-mass term of the type b^{ij} that can occur in the MSSM.

On dimensional grounds, we expect

$$M_1,\ M_2,\ M_3,\ a_u,\ a_d,\ a_e \sim m_{soft}$$
$$m_Q^2,\ m_L^2,\ m_{\bar{u}}^2,\ m_{\bar{d}}^2,\ m_{\bar{e}}^2,\ m_{H_u}^2,\ m_{H_d}^2,\ b \sim m_{soft}^2$$

with a characteristic mass scale m_{soft} that is not much larger than 1 TeV.

Unlike the supersymmetry-preserving part of the MSSM Lagrangian, the above $\mathcal{L}_{soft}^{MSSM}$ introduces many new parameters that were not present in the ordinary SM. A careful count reveals that there are 105 masses, phases and mixing in the MSSM Lagrangian that can not be rotated away by redefining the phases and flavor basis of the quark and lepton supermultiplets, and that have no counterpart in the ordinary SM. Thus, in principle, supersymmetry breaking appears to introduce a tremendous arbitrariness in the Lagrangian.

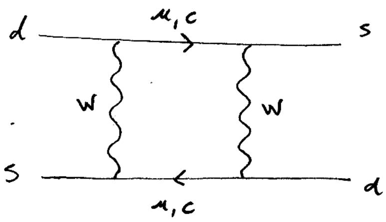
· Constraints on the supersymmetry breaking terms

In analyzing the phenomenological implications of the MSSM, we must deal not only with the search for new particles, but also with two classes of severe empirical constraints on processes involving known particles: the experimental upper bounds on various flavor non-conserving processes, and on various modes of CP non-conservation.

· Flavor changing processes

In the SM there is an automatic suppression of flavor changing processes like K^0-$\bar{K}^0$ oscillations and $K^0 \to \mu^+\mu^-$. This is due to the feature of this theory, that it is only the mass splittings of the quarks that prevent them from being defined so that each flavor is separately conserved. Hence the amplitude of these flavor-changing process must be proportional to several factors of small quark masses (this is known as the GIM mechanism, from the names of Glashow, Iliopoulos and Maiani, who pointed out this feature).

In the SM K^0-$\bar{K}^0$ oscillations, which are produced by operators in the effective low-energy Lagrangian like $(\bar{s}_L\gamma^\mu d_L)(\bar{s}_L\gamma_\mu d_L)$, can be produced by diagrams like

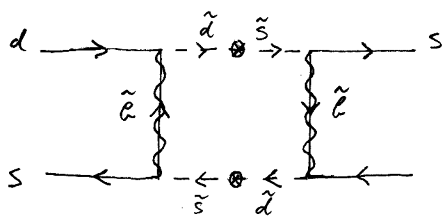


This gives the amplitude for $d_L\bar{s}_L \to s_L\bar{d}_L$; which is proportional to

$$\frac{g^4 \sin^2\theta_c \cos^2\theta_c}{m_W^4}\,(m_c - m_u)^2 ,$$

where θ_c is the Cabibbo angle that appears in the CKM matrix ($V_{ud} = \cos\theta_c$). Computing the K^0-$\bar{K}^0$ oscillations from this result one finds a prediction in good agreement with the experiments, it is then reasonable to require that the new physics contributions to the $d_L\bar{s}_L \to s_L\bar{d}_L$ process should be smaller than the SM ones.

The squarks contribute to this process through the diagrams of the type



If the superpartners of the d_L and s_L quarks are given by $\sum_i V_{di}\mathcal{D}_i$ and $\sum_i V_{si}\mathcal{D}_i$ of the squarks $\mathcal{D}_i$ of definite mass, we get an amplitude proportional to

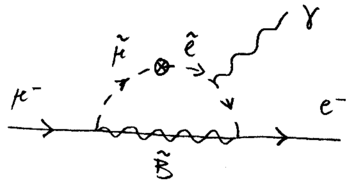
$$\frac{g_s^4}{\tilde{M}^6}\left(\sum_i V_{di}V_{si}^*\,\Delta M_i^2\right)^2 ,$$

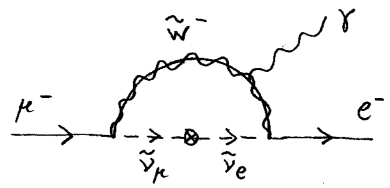
where $\tilde{M}$ is the largest of M_{squark} and m_{gluino}, and ΔM_i are the squark mass differences, while g_s is the strong coupling constant. Imposing that this amplitude is smaller than the SM one, we get the constraint

$$\left| \sum_i V_{di} V_{si}^* \frac{\Delta M_i^2}{\tilde{M}^2} \right| < 1.5 \cdot 10^{-3} \left(\frac{\tilde{M}}{100\,\mathrm{GeV}} \right).$$

The squark masses are unlikely to be much less than the gluino mass, so we can conclude that the squark masses are nearly degenerate, or the non-diagonal terms in the V_{ji} matrix must be small, or the squarks are heavier than about 10 TeV. This is one of the strongest constraints on the MSSM parameters.

Another constraint comes from the process $\mu \to e\gamma$. In the SM lepton flavor is automatically conserved, so that processes like $\mu \to e\gamma$ are absolutely forbidden. In the MSSM this process is generated by the diagrams





The branching ratio for this process can be estimated as

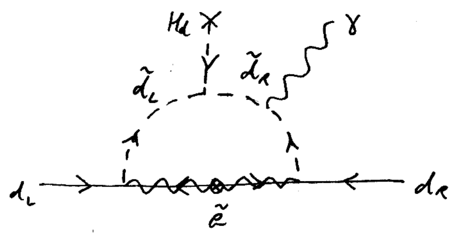
$$Br(\mu \to e\gamma) \sim 3 \cdot 10^{-4} \left(\frac{500\,\mathrm{GeV}}{\tilde{M}} \right)^4 \left(\frac{\Delta m^2}{\tilde{M}^2} \right)^2,$$

where $\tilde{M}$ is the mass scale of the superpartners, while Δm^2 is the mass splitting. Experimentally the branching ratio has an upper bound $Br(\mu \to e\gamma) < 4.9 \cdot 10^{-11}$, thus setting strong constraints on the superpartner masses and/or on the mass splittings.

- <u>CP violation</u>

The second important class of constraints provided by experimental data has to do with CP-violating effects, such as the electric dipole moments of the neutron and electron. These effects are rather small in the SM. This is because all CP-violating phases in the mass matrix of quarks and leptons and their interaction with gauge bosons could be absorbed into the definition of the quark and lepton fields if there were only two generations of quarks and leptons, and although there is a third generation, its mixing with the first two generations is quite small. The electric dipole moment of the neutron in the SM is consequently expected to be less than about 10^{-30} e cm, well below the experimental upper bound, $0.87 \cdot 10^{-15}$ e cm.

In contrast, the parameters of the MSSM include dozens of CP-violating phases. For example the amplitude



gives rise to an electric dipole moment for the d quark, and hence for the neutron. In the above diagram the X denotes an insertion of the H_d VEV, which arises after electroweak symmetry breaking. Since the electric dipole moment is a CP-violating quantity, the amplitude needs a non-trivial complex phase which can be supplied by the soft susy-breaking parameters. If we call the overall phase δ then the electric dipole moment is approximately given by

$$\mathcal{M}_{EDM} \sim \frac{g^2}{16\pi^2}\,\frac{e\langle H_d\rangle (a_d)_{11}\,\delta}{\tilde{M}^2}\,.$$

The experimental bound on the electric dipole moment of the neutron translates into the bound

$$(a_d)_{11}\,\delta\left(\frac{500\,\mathrm{GeV}}{\tilde{M}}\right)^2 \lesssim 5\cdot 10^{-7},$$

with $(a_d)_{11}$ in GeV units.

· Soft supersymmetry breaking universality

All the potentially dangerous flavor-changing and CP-violating effects in the MSSM can be avoided if one assumes that susy breaking is suitably "universal". If the squark and slepton squared-mass matrices are flavor-blind

$$m^2_{\tilde{Q}} = m^2_{\tilde{Q}}\,\mathbb{1}\,,\quad m^2_{\tilde{u}} = m^2_{\tilde{u}}\,\mathbb{1}\,,\quad m^2_{\tilde{d}} = m^2_{\tilde{d}}\,\mathbb{1}\,,\quad m^2_{\tilde{L}} = m^2_{\tilde{L}}\,\mathbb{1}\,,\quad m^2_{\tilde{e}} = m^2_{\tilde{e}}\,\mathbb{1}\,, \qquad (*)$$

then all squark and slepton mixing angles are rendered trivial, because squarks and sleptons with the same quantum numbers will be degenerate in mass and can be rotated into each other. Susy contributions to flavor-changing processes will be very small in such a limit, up to mixing induced by a_u, a_d, a_e. With the further assumption that the $(\text{scalar})^3$ couplings are proportional to the corresponding Yukawa coupling matrix

$$a_u = A_{u0}\, y_u\,,\qquad a_d = A_{d0}\, y_d\,,\qquad a_e = A_{e0}\, y_e\,, \qquad (**)$$

only the squarks and sleptons of the third family can have large $(\text{scalar})^3$ couplings. Finally, one can avoid large CP-violating effects by assuming that the soft parameters do not introduce new complex phases. This is automatic for $m^2_{H_u}$ and $m^2_{H_d}$, and for $m^2_{\tilde{Q}}$, $m^2_{\tilde{u}}$, etc. if eq. $(*)$ is assumed; if they were not real numbers, the Lagrangian would not be real. One can also fix μ in the superpotential and b to be real, by appropriate phase rotations of fermions and scalar components of the H_u and H_d supermultiplets. If one then assumes

$$\arg(M_1),\ \arg(M_2),\ \arg(M_3),\ \arg(A_{u0}),\ \arg(A_{d0}),\ \arg(A_{e0}) = 0 \text{ or } \pi \qquad (***)$$

then the only CP-violating phase in the theory will be the usual CKM phase found in the ordinary Yukawa couplings.

Conditions $(*)$, $(**)$ and $(***)$ implement the hypothesis of soft supersymmetry-breaking universality.

NOTE. Often the universality conditions on the soft breaking terms are imposed at a high-energy scale (usually the coupling unification scale $\sim 10^{16}$ GeV). The soft terms at low-energy are obtained by a renormalization group evolution.

There are other possible types of explanations for the suppression of flavor violation in the MSSM, that could replace the universality hypothesis. One possibility is the so called "alignment" hypothesis, that is the idea that the squark squared-mass matrices do not have the flavor blindness in eq. (*), but are arranged in flavor space to be aligned with the relevant Yukawa matrices in just a way as to avoid large flavor-changing effects. The alignment models typically require rather special flavor symmetries.

· gauge coupling unification

Now we consider an application of supersymmetry in one context in which the mechanism for the breakdown of supersymmetry is relatively unimportant, and in which supersymmetry has scored a great empirical success.

If the $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge group of the SM is embedded in a simple group G that has the known quarks and leptons (plus perhaps some $SU(3)_c \times SU(2)_L \times U(1)_Y$-neutral fermions) as a representation, then at energies at or above the scale M_X at which G is spontaneously broken, the $SU(3)_c \times SU(2)_L \times U(1)_Y$ coupling constants will be related by

$$g_s^2 = g^2 = \frac{5}{3} g'^2 \quad \text{at energies} \gtrsim M_X .$$

This, for example happens if we embed the SM group into $SU(5)$ or $SO(10)$. At energies far below M_X, these couplings are seriously affected by renormalization corrections. If measured at a scale $\mu < M_X$, the couplings will have values $g_s^2(\mu)$, $g^2(\mu)$, $g'^2(\mu)$ governed by the one-loop renormalization group equations

$$\mu \frac{d}{d\mu} g_i(\mu) = \beta_i(g_i(\mu)) \equiv \frac{1}{16\pi^2} b_i g_i^3 .$$

This implies that $\alpha_i^{-1} \equiv (g_i^2/4\pi)^{-1}$ run linearly with the energy at one-loop:

$$\mu \frac{d}{d\mu} \alpha_i^{-1} = -\frac{b_i}{2\pi} .$$

Since M_X will turn out to be many orders of magnitude larger than the energies accessible with today's accelerators, it seems reasonable to suppose that supersymmetry is unbroken over most of the range below M_X, in which case all the superpartners need to be included in the computation of the $\beta(g_i)$ functions.

In the SM the β function coefficients are

$$\begin{cases} b(g') = +\frac{41}{6} \\ b(g) = -\frac{19}{6} \\ b(g_s) = -7 \end{cases}$$

With these values the coupling constants become almost equal at an energy scale $\mu \sim 10^{13} - 10^{15}$ GeV, but they do not exactly unify.

On the other hand in the MSSM we have

$$\begin{cases} b(g') = 10 + \dfrac{n_S}{2} \\ b(g) = + \dfrac{n_S}{2} \\ b(g_s) = -3 \end{cases}$$

where n_S is the number of Higgs doublets in the model.

We can express the electroweak couplings in terms of the weak mixing angle θ and the positron charge e:

$$g(m_Z) = + e(m_Z)/\sin\theta \quad , \quad g'(m_Z) = + e(m_Z)/\cos\theta \, .$$

If we assume unification at a scale M_X, we can solve the renormalization group expressions for $g'(m_Z)$, $g(m_Z)$ and $g_s(m_Z)$ in order to find $\ln(M_X/m_Z)$ and $\sin^2\theta$ as functions of $e(m_Z)$ and $g_s(m_Z)$ (exercise):

$$\sin^2\theta = \frac{18 + 3n_S + \left(e^2(m_Z)/g_s^2(m_Z)\right)(60 - 2n_S)}{108 + 6n_S} \, ,$$

$$\ln\left(\frac{M_X}{m_Z}\right) = \left(\frac{8\pi^2}{e^2(m_Z)}\right)\left(\frac{1 - 8e^2(m_Z)/(3g_s^2(m_Z))}{18 + n_S}\right) .$$

Using the values

$$\frac{e^2(m_Z)}{4\pi} = \frac{1}{128}$$

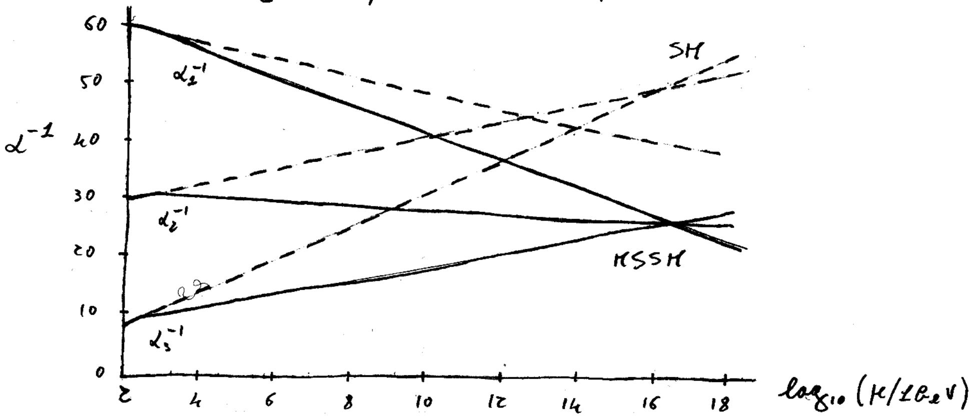
$$\frac{g_s^2(m_Z)}{4\pi} = 0.118$$

$$m_Z = 91.19 \text{ GeV}$$

we get

n_S	$\sin^2\theta$	M_X (GeV)
0	0.203	$8.7 \cdot 10^{17}$
2	0.231	$2.2 \cdot 10^{16}$
4	0.253	$1.1 \cdot 10^{15}$

Remarkably, the value $n_S = 2$ for the simplest plausible theory yields a value $\sin^2\theta = 0.231$ which is in perfect agreement with the experimentally observed value $\sin^2\theta = 0.23$. The value of M_X is 20 times greater than the one calculated in this way in non-susy GUT theories, leading to a decrease by a factor 20^{-4} in the rate for proton decay processes like $p \to \pi^0 + e^+$, thus removing a conflict with the experimental non observation of such processes.



• Electroweak symmetry breaking and the Higgs bosons

In the MSSM, the description of electroweak symmetry breaking is slightly complicated by the fact that there are two complex Higgs doublets $H_u = (H_u^+, H_u^0)$ and $H_d = (H_d^0, H_d^-)$ rather than just one in the ordinary SM. The classical scalar potential for the Higgs scalar fields in the MSSM is given by

$$V = (|\mu|^2 + m_{H_u}^2)(|H_u^0|^2 + |H_u^+|^2) + (|\mu^2| + m_{H_d}^2)(|H_d^0|^2 + |H_d^-|^2)$$
$$+ [b(H_u^+ H_d^- - H_u^0 H_d^0) + h.c.]$$
$$+ \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2)^2 + \frac{1}{2} g^2 |H_u^+ H_d^{0*} + H_u^0 H_d^{-*}|^2 .$$

The terms proportional to $|\mu^2|$ come from F-terms, while the terms proportional to g^2 and g'^2 are the D-term contributions. Finally, the terms proportional to $m_{H_u}^2$, $m_{H_d}^2$ and b are just a rewriting of the terms of soft susy breaking.

NOTE. The full scalar potential also includes many terms involving the squarks and slepton fields. We can ignore them since they do not get VEV's (which would break the gauge symmetries) because they have large positive squared masses.

We must now require that the minimum of the potential breaks the electroweak group to electromagnetism: $SU(2)_L \times U(1)_Y \to U(1)_{EM}$. We can use the freedom to make $SU(2)_L$ gauge transformations to set $\langle H_u^+ \rangle = 0$ at the minimum of the potential. If we look for a stable minimum along the charged directions we find

$$\left.\frac{\partial V}{\partial H_u^+}\right|_{\langle H_u^+ \rangle = 0} = b H_d^- + \frac{g^2}{2} H_d^{0*} H_d^- H_u^{0*}$$

which will not vanish for non-zero H_d^- for generic values of the parameters. This implies that $\langle H_d^- \rangle = 0$ and $U(1)_{EM}$ is unbroken. We are left with

$$V = (|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu^2| + m_{H_d}^2)|H_d^0|^2 - (b H_u^0 H_d^0 + h.c.)$$
$$+ \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2)^2 .$$

By a phase redefinition of H_u or H_d we can set b to be real. Then, at the minimum of the potential, $\langle H_u^0 \rangle$ and $\langle H_d^0 \rangle$ have opposite phases, and, since they have opposite $U(1)_Y$ hypercharges, we can set these phases to zero by a $U(1)_Y$ gauge transformation. Thus CP symmetry is not spontaneously broken in the MSSM.

In order to have a sensible theory we need to impose that V is bounded from below. Note that this would be automatically true in a susy-invariant theory, but it is not so when we introduce soft susy-breaking terms. The scalar quartic interaction in V stabilizes the potential for all the directions except when $H_u^0 = H_d^0$. So we need the quadratic part to be positive along this direction. This requirement gives

$$2b < 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2 .$$

Moreover we need to require that one linear combination of H_u^0 and H_d^0 has a negative squared mass at the origin:

$$b^2 > (|\mu|^2 + m_{H_u}^2)(|\mu|^2 + m_{H_d}^2).$$

Notice that, if $m_{H_u}^2 = m_{H_d}^2$, the above constraints can not both be satisfied. In models with minimal supergravity or gauge mediated susy breaking one usually finds $m_{H_u}^2 = m_{H_d}^2$ at the susy-breaking scale. But then the renormalization group evolution for $m_{H_u}^2$ naturally pushes it to negative or small values $m_{H_u}^2 < m_{H_d}^2$ at the electroweak scale. So in these models electroweak symmetry breaking is induced by radiative corrections.

We can now study the electroweak-breaking minimum. Let's write

$$\langle H_u^0 \rangle = \frac{v_u}{\sqrt{2}} \quad \text{and} \quad \langle H_d^0 \rangle = \frac{v_d}{\sqrt{2}}.$$

Such VEV's break electroweak symmetry, hence giving W and Z masses

$$M_W^2 = \frac{1}{4} g^2 v^2,$$
$$M_Z^2 = \frac{1}{4}(g^2 + g'^2) v^2,$$

where

$$v^2 = v_u^2 + v_d^2 \simeq (246\ \text{GeV})^2.$$

We can rewrite the ratio of the VEV's in terms of an angle β

$$s_\beta \equiv \sin\beta \equiv \frac{v_u}{v}, \quad c_\beta \equiv \cos\beta \equiv \frac{v_d}{v},$$

with $0 < \beta < \pi/2$. The VEV's can be related to the parameters in the potential by imposing the minimum conditions $\partial V/\partial H_u^0 = \partial V/\partial H_d^0 = 0$, which give

$$|\mu|^2 + m_{H_u}^2 = b\cot\beta + \frac{M_Z^2}{2}\cos 2\beta,$$
$$|\mu|^2 + m_{H_d}^2 = b\tan\beta - \frac{M_Z^2}{2}\cos 2\beta.$$

The Higgs scalar consists of eight real scalar degrees of freedom. When the electroweak symmetry is broken, three of them are the would-be Goldstone bosons which are eaten by the Z^0 and $W^\pm$. This leaves five degrees of freedom A^0, $H^\pm$, h^0 and H^0. The h^0 and H^0 are CP even and the A^0 is CP odd. It is convenient to shift the fields by these VEV's

$$H_u^0 = \frac{v_u}{\sqrt{2}} + \mathcal{H}_u^0,$$
$$H_d^0 = \frac{v_d}{\sqrt{2}} + \mathcal{H}_d^0,$$

then we can read off the mass terms for the various physical Higgs components.

For the imaginary parts of the neutral fields we get

$$V \supset (\mathrm{Im}\,\mathcal{H}_u^0, \mathrm{Im}\,\mathcal{H}_d^0) \begin{pmatrix} b\cot\beta & b \\ b & b\tan\beta \end{pmatrix} \begin{pmatrix} \mathrm{Im}\,\mathcal{H}_u^0 \\ \mathrm{Im}\,\mathcal{H}_d^0 \end{pmatrix}.$$

Diagonalizing, we find the two mass eigenstates:

$$\begin{pmatrix} \pi^0 \\ A^0 \end{pmatrix} = \sqrt{2} \begin{pmatrix} s_\beta & -c_\beta \\ c_\beta & s_\beta \end{pmatrix} \begin{pmatrix} \mathrm{Im}\, H_u^0 \\ \mathrm{Im}\, H_d^0 \end{pmatrix}.$$

π^0 is massless and is the would-be Goldstone eaten by the Z^0, while A^0 has a mass

$$m_A^2 = \frac{b}{s_\beta c_\beta} = 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2.$$

Considering the charged components

$$V \supset (H_u^{+*}, H_d^-) \begin{pmatrix} b\cot\beta + M_W^2 c_\beta^2 & b + M_W^2 c_\beta s_\beta \\ b + M_W^2 c_\beta s_\beta & b\tan\beta + M_W^2 s_\beta^2 \end{pmatrix} \begin{pmatrix} H_u^+ \\ H_d^{-*} \end{pmatrix},$$

and the mass eigenstates are

$$\begin{pmatrix} \pi^+ \\ H^+ \end{pmatrix} = \begin{pmatrix} s_\beta & -c_\beta \\ c_\beta & s_\beta \end{pmatrix} \begin{pmatrix} H_u^+ \\ H_d^{-*} \end{pmatrix},$$

where $\pi^- = \pi^{+*}$ are the would-be Goldstones eaten by the $W^\pm$ and $H^- = H^{+*}$. So the mass of the charged Higgs is

$$m_{H^\pm}^2 = m_A^2 + M_W^2.$$

Finally, for the real parts of the neutral fields we have

$$V \supset (\mathrm{Re}\, H_u^0, \mathrm{Re}\, H_d^0) \begin{pmatrix} b\cot\beta + M_Z^2 s_\beta^2 & -b - M_Z^2 c_\beta s_\beta \\ -b - M_Z^2 c_\beta s_\beta & b\tan\beta + M_Z^2 c_\beta^2 \end{pmatrix} \begin{pmatrix} \mathrm{Re}\, H_u^0 \\ \mathrm{Re}\, H_d^0 \end{pmatrix},$$

which has mass eigenstates

$$\begin{pmatrix} h^0 \\ H^0 \end{pmatrix} = \sqrt{2} \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \mathrm{Re}\, H_u^0 \\ \mathrm{Re}\, H_d^0 \end{pmatrix},$$

with masses

$$m_{h,H}^2 = \frac{1}{2}\left(m_A^2 + M_Z^2 \mp \sqrt{(m_A^2 - M_Z^2)^2 + 4 M_Z^2 m_A^2 \sin^2 2\beta} \right),$$

and the mixing angle α is determined by

$$\frac{\sin 2\alpha}{\sin 2\beta} = -\frac{m_A^2 + m_Z^2}{m_H^2 - m_h^2}, \quad \frac{\cos 2\alpha}{\cos 2\beta} = -\frac{m_A^2 - m_Z^2}{m_H^2 - m_h^2}.$$

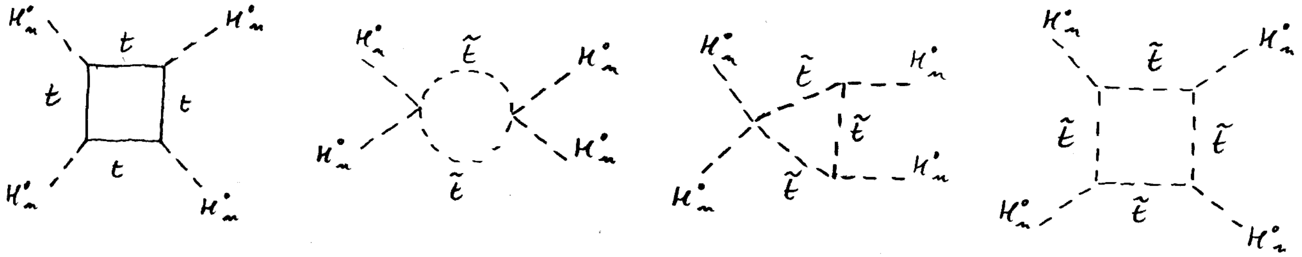
Note that m_A, $m_{H^\pm}$, and m_H grow as $b \to \infty$, but m_h is maximized at $m_A \to \infty$, so at tree level there is an upper bound on the Higgs mass

$$m_h < |\cos 2\beta| M_Z = \frac{g^2 + g'^2}{4} |v_d^2 - v_u^2|.$$

which is essentially ruled out by experiment.

For $m_A \gg M_Z$, then A^0, H^0 and $H^\pm$ are much heavier than h^0, forming a nearly degenerate isospin doublet. In this decoupling limit, the angle α is fixed to be approximately $\beta - \pi/2$, and h^0 has SM couplings to quarks, leptons and gauge bosons.

The upper bound on the Higgs mass is relaxed if we also include radiative corrections. One can compute the contribution to the Higgs mass by evaluating the one-loop corrections to the quartic coupling of the potential. The largest contribution comes from top - stop loops. If the stops are heavy with respect to the top the loop diagrams do not exactly cancel. Some of the most important diagrams are



This leads to a shift in the physical Higgs mass

$$\Delta(m_h^2) = \frac{3}{4\pi^2}\, v^2 y_t^4 \sin^2\beta \ln \frac{m_{\tilde t_1} m_{\tilde t_2}}{m_t^2} \approx \frac{(90\,GeV)^2}{\sin^2\beta}$$

valid for not too small $\sin\beta$ (otherwise the top Yukawa coupling will be non-perturbative). In this way we get a weaker bound

$$\underline{m_h \lesssim 130\ GeV.}$$

• After electroweak symmetry breaking the superpartners with the same $SU(3)_c$ and electroweak quantum numbers can mix to give new mass eigenstates. In particular all the squarks (sleptons) with the same electric charge can mix among themselves. For the gauginos we get that:

- the gluinos can not mix with other states;
- the charged winos ($\tilde W^\pm$) mix with the charged Higgsinos ($\tilde H_u^+$ and $\tilde H_d^-$) giving rise to the <u>charginos</u> $\tilde C_1^\pm$, $\tilde C_2^\pm$;
- the neutral wino ($\tilde W^0$) and the bino ($\tilde B^0$) mix with the neutral Higgsinos ($\tilde H_u^0$, $\tilde H_d^0$) giving rise to the <u>neutralinos</u> ($\tilde N_1$, $\tilde N_2$, $\tilde N_3$, $\tilde N_4$).

<u>NOTE</u>. For possible collider signatures of the MSSM see for example the review by S.P. Martin.