QFT III

The aim of the course is to give a comprehensive introduction to Supersymmetry. We will discuss the theoretical basis of SUSY and then we will see some phenomenological application of SUSY in the physics beyond the Standard Model (MSSM).

I. INTRODUCTION

Modern physics, and in particular QFT, is primarily based on SYMMETRIES, which are used to build the fundamental structure of our theories.

Many examples in QFT:

- Lorentz and Poincaré
- Global symmetries (eg. lepton number, baryon number)
- Gauge symmetries (eg. $U(1)_{em}$ in QED
  $SU(2)_L \times U(1)_Y$ in SM
  $SU(3)_c$ in QCD)

All these symmetries have a particular aspect in common: they relate particles with the SAME SPIN (of course Lorentz is special because it "contains" the "definition" of spin).

Theoretical question: Can we introduce symmetries which relate particles of different spin (a so-called supersymmetry)?

· We start to answer this question by giving a look to the historical development of supersymmetry.

· The non-relativistic quark model (Weinberg, vol III chapter 24)

The first attempts to use symmetries which connect particles of different spin were made in the early 1960s in the context of the theories that describe hadrons.

Hadrons were identified with the bound states in the non-relativistic quark model. This is a quantum theory whose fundamental objects are the light quarks:

$$u\,,\ d\,,\ s\,.$$

The theory is based on a global $SU(3)$ symmetry under which the quarks transform in the fundamental representation. This is a flavour symmetry.

NOTE: Notice that the SU(3) symmetry of the non-relativistic quark model is NOT the colour $SU(3)_c$ symmetry of QCD. The colour symmetry is a gauge symmetry (and it implies the presence of the gluons), the flavour SU(3) is just a global symmetry!

If we consider a system of N quarks, the Hamiltonian of the system could depend on the positions and momenta as well as

- the spins: given by the operators $\sigma_i^{(n)}$ which act on the n-th quark ($i=1,2,3$ give the Pauli matrices);
- the flavour: given by the operators $\lambda_A^{(n)}$ which act on the n-th quark ($A=1,..,8$ give the Gell-Mann SU(3) matrices).

According to the invariance properties of the Hamiltonian we can have different symmetry groups for the theory.

The largest symmetry we can have is $SU(6)^N$ which is obtained if the Hamiltonian is completely independent of spin and flavour. The generators of such invariance are

$$\begin{cases} S_i^{(n)} = \frac{1}{2}\sigma_i^{(n)} \\ T_A^{(n)} = \frac{1}{2}\lambda_A^{(n)} \\ R_{iA}^{(n)} \equiv \frac{1}{2}\sigma_i^{(n)}\lambda_A^{(n)} . \end{cases}$$

In some case a smaller symmetry was assumed by including spin- and flavour-dependent two-body interactions

$$H^{(nm)} \propto \left[1 + \sum_i \sigma_i^{(n)}\sigma_i^{(m)}\right]\left[\frac{2}{3} + \sum_A \lambda_A^{(n)}\lambda_A^{(m)}\right] ,$$

in this case only an SU(6) symmetry survives, whose generators are

$$\begin{cases} S_i \equiv \sum_n S_i^{(n)} \\ T_A \equiv \sum_n T_A^{(n)} \\ R_{iA} \equiv \sum_n R_{iA}^{(n)} . \end{cases}$$

All these symmetries are only approximate, and they are broken in Nature.

An interesting point is that the SU(6) symmetry relates particles of different spins (although it relates bosons of different spins or fermions of different spin, but not bosons with fermions), in a theory which is not spin-independent. However in Nature this SU(6) symmetry does not seem to be better satisfied than the complete spin and flavour independence.

The SU(6) symmetry is a symmetry of a NON-relativistic model and the various attempts to generalize it to a relativistic theory failed.
It was also proved that such a generalization is impossible under some (mild) assumptions: this is the Coleman-Mandula theorem.

Theorem (Coleman-Mandula, 1967). The only possible Lie algebra of symmetry generators consists of the Poincaré generators, together with possible internal symmetry generators, which commute with the Poincaré generators, and act on physical states by multiplying them with spin- and momentum-independent Hermitian matrices. This is true under the assumptions

1) For any M there are only a finite number of particle types with mass less than M.
2) Any two-particle state undergoes some reaction at almost all energies (except perhaps an isolated set).
3) The amplitudes for elastic two-body scattering are analytic functions of the scattering angle at almost all energies and angles.

NOTE. In theories with only massless particles the most general Lie algebra of symmetries is given by the conformal group plus internal symmetries.

NOTE 2. See Weinberg, vol. III chapter 24 appendix B for a proof.

· String theory.

After the non-relativistic quark model a symmetry which related particles of different spin appeared in string theory, which, at that time, was an attempt to describe strong interactions (late 1960s).

String theory describes the dynamics of 1-dimensional objects:

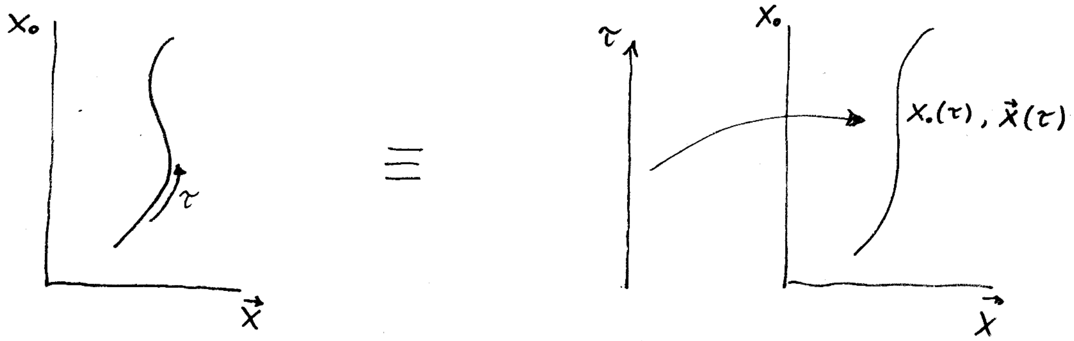
open string 

closed string 

· The action for a string.

To find out the symmetries of the string we need to write down its action.
We will do this by using a procedure similar to the one for point-particles.

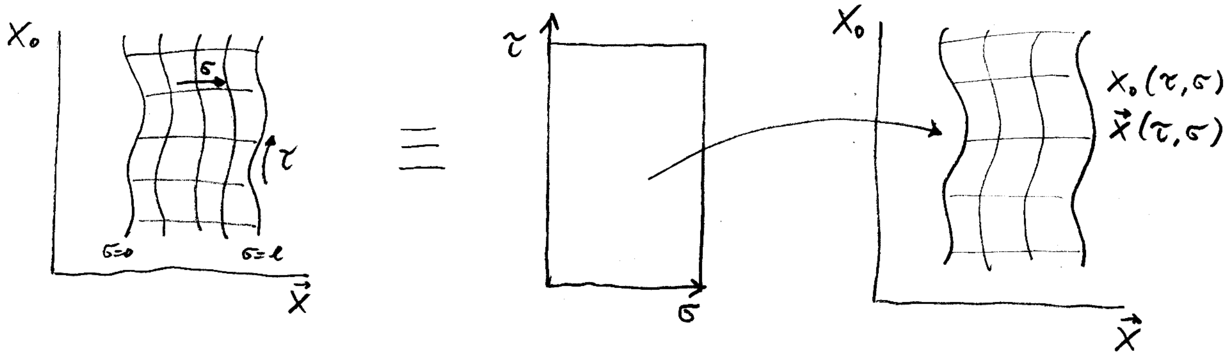
· world-line for a massless point particle



the action is

$$S_{pp} = \int d\tau \, \eta_{\mu\nu} \frac{\partial X^\mu}{\partial \tau} \frac{\partial X^\nu}{\partial \tau} .$$

we can proceed analogously for the string



The <u>bosonic string action</u> for d bosonic fields is $(\mu = 0, 1, \ldots, d-1)$

$$I[X] = \frac{T}{2} \int d\sigma \int d\tau \, \eta_{\mu\nu} \left[\frac{\partial X^\mu}{\partial \tau} \frac{\partial X^\nu}{\partial \tau} - \frac{\partial X^\mu}{\partial \sigma} \frac{\partial X^\nu}{\partial \sigma} \right]$$

$$= T \int d\sigma^+ \int d\sigma^- \, \eta_{\mu\nu} \frac{\partial X^\mu}{\partial \sigma^+} \frac{\partial X^\nu}{\partial \sigma^-} .$$

where T is a constant (the "string tension") and

$$\sigma^\pm \equiv \tau \pm \sigma$$

are the 'light-cone' coordinates.

· The string action has a <u>global conformal invariance</u> under

$$\underline{\sigma^\pm \to f^\pm(\sigma^\pm)}$$

with $f^\pm$ a pair of independent arbitrary functions.

One can also introduce fermions in the string action. We add d fermion pairs $\psi_1^\mu(\sigma,\tau)$, $\psi_2^\mu(\sigma,\tau)$ with the action

$$I[X,\psi] = \int d\sigma^+ \int d\sigma^- \left[T \frac{\partial X^\mu}{\partial \sigma^+}\frac{\partial X_\mu}{\partial \sigma^-} + i\psi_2^\mu \frac{\partial}{\partial \sigma^+}\psi_{2\mu} + i\psi_1^\mu \frac{\partial}{\partial \sigma^-}\psi_{1\mu} \right].$$

This action has still a conformal invariance if we choose the transformation rules

$$\psi_1^\mu \to \left(\frac{df^+}{d\sigma^+}\right)^{-1/2}\psi_1^\mu \quad , \quad \psi_2^\mu \to \left(\frac{df^-}{d\sigma^-}\right)^{-1/2}\psi_2^\mu .$$

· For appropriate boundary conditions the theory has also an extra invariance which exchanges fermions and bosons

$$\begin{cases} \delta\psi_2^\mu(\sigma^+,\sigma^-) = i\,T\,\alpha_2(\sigma^-)\,\dfrac{\partial}{\partial\sigma^-}\,X^\mu(\sigma^+,\sigma^-) \\ \delta\psi_1^\mu(\sigma^+,\sigma^-) = i\,T\,\alpha_1(\sigma^+)\,\dfrac{\partial}{\partial x^+}\,X^\mu(\sigma^+,\sigma^-) \\ \delta X^\mu(\sigma^+,\sigma^-) = \alpha_2(\sigma^-)\,\psi_2^\mu(\sigma^+,\sigma^-) + \alpha_1(\sigma^+)\,\psi_1^\mu(\sigma^+,\sigma^-) \end{cases}$$

where $\alpha_{1,2}$ are infinitesimal <u>fermionic</u> functions (like the Grassmann variables). This symmetry and the conformal symmetry form a so called <u>superconformal symmetry</u> for the 2-dimensional theory.

· What about the Coleman - Mandula theorem? The symmetry generators do <u>NOT</u> form a Lie algebra, SUSY generators are <u>fermionic</u> and satisfy <u>anticommutation relations</u>.

· The Wess - Zumino model

Later the notion of supersymmetry was promoted to a spacetime supersymmetry by Wess and Zumino, who constructed the first examples of supersymmetric models in 4 dimensions.

We will discuss this models in details during the course.

· Supersymmetry in strings was also promoted to a 10-dimensional spacetime supersymmetry by Gliozzi, Scherk and Olive by imposing suitable periodicity conditions for the fields.

<u>NOTE</u>. Supersymmetry was also discovered by some russian authors before the models by Wess and Zumino, however their papers received little or no attention by the physics community.

· Phenomenological reasons for supersymmetry.

Nowadays SUSY is also popular for phenomenological reasons related to the physics beyond the Standard Model (SM).

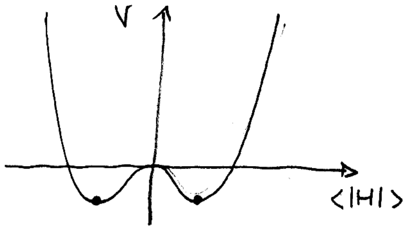
· Naturalness in the SM

The SM is a gauge theory based on the gauge group $SU(2)_L \times U(1)_Y$. In Nature we do not observe long range interactions which respect the $SU(2)_L \times U(1)_Y$ group, so we know that this symmetry must be broken to the electromagnetic group $U(1)_{EM}$.

In the SM this breaking occurs spontaneously and is generated by a scalar field: the Higgs boson H. The Lagrangian for the Higgs contains a potential term

$$V = m_H^2 |H|^2 + \lambda |H|^4 .$$

If $m_H^2 < 0$ (and $\lambda > 0$) the Higgs gets a vacuum expectation value (VEV) determined by the position of the minimum of the potential:



$$\underline{\langle H \rangle = \sqrt{-\frac{m_H^2}{2\lambda}}} .$$

The value of the Higgs VEV is a very important quantity in the SM, because it determines the values of the masses of all the SM fields. One gets experimentally

$$\underline{\langle H \rangle = 246 \text{ GeV}} .$$

This means that a "natural" value for the m_H^2 term in the Lagrangian (and thus for the Higgs mass) is of order $\sim -(100 \text{ GeV})^2$, unless we allow for unnaturally large values of λ (which would also drive the theory in a non-perturbative regime).

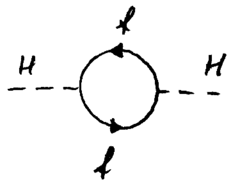
But there is also a second constraint on the Higgs mass which comes from the unitarity of the WW scattering. The $WW \to WW$ cross section, in the absence of the Higgs field, grows with the energy and violates unitarity at

$$\underline{E \sim 1 \text{ TeV}} .$$

The addition of an Higgs field cancels part of the $WW \to WW$ cross section, thus allowing the theory to remain unitary up to very large energies. For this to happen the Higgs must be light enough, namely $\underline{m_H \lesssim 1 \text{ TeV}}$.

The problem with this picture is the fact that the Higgs mass term receives enormous quantum corrections from the loop effects of all the particles that couple (directly or indirectly) to the Higgs field.

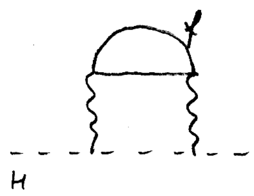
For example if we consider a fermion with mass m_f which couples to the Higgs as $-\lambda_f H \bar{f} f$ we get in a cut-off regularization



$$\Rightarrow \quad \Delta m_H^2 = -\frac{|\lambda_f|^2}{8\pi^2}\left[\Lambda_{UV}^2 - 3 m_f^2 \ln\left(\frac{\Lambda^2+m_f^2}{m_f^2}\right) + \dots\right]$$

The cut-off scale Λ_{UV} can be interpreted as the energy scale at which some new physics appears which can correct the behaviour of the loop integral.

Notice that similar corrections come also from particles which couple to the Higgs only indirectly (eg. through gauge interactions)



$$\Rightarrow \quad \Delta m_H^2 \sim \left(\frac{g^2}{16\pi^2}\right)^2 \cdot \Lambda_{UV}^2 + \dots$$

The physical Higgs mass is given by the sum of the bare mass $(m_H^2)_B$ and the quantum corrections:

$$(m_H^2)_{phys} = (m_H^2)_B + \Delta m_H^2$$

and if we assume that the cut-off of the SM is not very low we need a large tuning of $(m_H^2)_B$ in order to cancel the correction from Δm_H^2 and get a light Higgs ($m_H^2 \lesssim 1$ TeV).

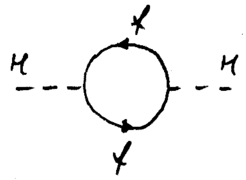
If we assume, for example, that the SM cut-off is of the order of the Planck mass $\Lambda_{UV} \sim M_P \simeq 10^{18}$ GeV, in order to get $m_H^2 \lesssim 1$ TeV, we would need to tune $(m_H^2)_B$ at the $\sim 10^{-30}$ level.

This situation seems unnatural and would mean that a small (in percent) variation of the parameters of the theory would produce huge differences in the physical predictions at low-energy.

This kind of fine-tuning problem related to the huge energy difference between an high cut-off scale (eg. the Planck scale) and the electroweak scale (~ 1 TeV) is called the "Hierarchy problem".

Notice that even if we use a different regularization procedure (and we do not want to associate Λ_{UV} to a physical scale) we still have some troubles with the Higgs mass term.

For example in dimensional regularization we get



$$\Rightarrow \quad \Delta m_H^2 \sim \frac{\lambda_f^2}{16\pi^2} m_f^2 \left[\frac{1}{\varepsilon} + \left(a + b \ln \frac{m_f^2}{\mu^2} \right) + \cdots \right]$$

This means that the Higgs mass term is quadratically sensitive to the mass of the heaviest particles which couple to the Higgs, or, in other words, that the UV physics can not be decoupled from the Higgs sector.

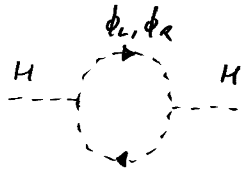
· Supersymmetry and the cancellation of divergences

Let's see what happens if we include some scalars in the theory. We consider two scalars ϕ_L and ϕ_R whose Lagrangian is

$$\mathcal{L}_S = -\frac{\lambda}{2} H^2 \left(|\phi_L|^2 + |\phi_R|^2 \right) - H \left(\mu_L |\phi_L|^2 + \mu_R |\phi_R|^2 \right) - m_L^2 |\phi_L|^2 - m_R^2 |\phi_R|^2$$

We get a new contribution to the Higgs self-energy:

(a)


(b)


the contribution of these diagrams is

$$\Delta m_H^2\Big|_a = \frac{\lambda}{16\pi^2} \left[2\Lambda_{UV}^2 - m_L^2 \ln\left(\frac{\Lambda_{UV}^2 + m_L^2}{m_L^2} \right) - m_R^2 \ln\left(\frac{\Lambda_{UV}^2 + m_R^2}{m_R^2} \right) + \cdots \right]$$

$$\Delta m_H^2\Big|_b = -\frac{1}{16\pi^2} \left[\mu_L^2 \ln\left(\frac{\Lambda^2 + m_L^2}{m_L^2} \right) + \mu_R^2 \ln\left(\frac{\Lambda^2 + m_R^2}{m_R^2} \right) + \cdots \right]$$

If we choose

$$\lambda = |\lambda_f|^2 \qquad (*)$$

we cancel the quadratic divergences in Δm_H^2 from the fermion loop.

Moreover if we also impose

$$\begin{cases} m_f = m_L = m_R \\ \mu_L^2 = \mu_R^2 = 2\lambda m_f^2 \end{cases} \qquad (**)$$

we also cancel the logarithmic pieces in the Higgs mass corrections.

Of course, if we impose these conditions by hand, we are just using a fine-tuning in the parameters of the model to eliminate the corrections to the Higgs mass. This means that the fine-tuning problem is only shifted to other parameters but not eliminated.

On the other hand, if we have a symmetry which relates the parameters of the fermionic and the bosonic Lagrangian, then we really solve the fine-tuning issue. This symmetry must relate bosons and fermions, so the only possibility is that it is supersymmetry. As we will see, supersymmetry implies exactly the relations in eqs. (*) and (**), so it is a good candidate for the solution of the Hierarchy problem.

· Literature

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