Quantum Field Theory III

HS 10, Exercise sheet 9

Due date: 24.11.2010

Exercise 1:
In this exercise we will explore the supersymmetric Higgs mechanism.
Look at a model with three chiral superfields Φ_0, Φ_+ and Φ_- with quantum numbers 0, $+1$ and -1 under global $U(1)$ symmetry. The superpotential is given by

$$W(\Phi) = \frac{1}{2} m \Phi_0^2 + \mu \Phi_+ \Phi_- + \lambda \Phi_0 + g \Phi_0 \Phi_+ \Phi_-.$$

a) Show that the superpotential is invariant global $U(1)$ symmetry.

b) Supersymmetry remains unbroken, if the vacuum is supersymmetric, in terms of the scalar potential this means $V = 0$. Show that there are two sets of vacua for the scalar components of the superfields, which preserve supersymmetry. Show that one set of vacua leaves $U(1)$-symmetry invariant whereas the other does not.

c) Calculate the masses of the chiral states and verify that bosons and their fermionic superpartners have the same mass.

d) Now we introduce gauge invariance in this theory: The complete Lagrangian consists of the kinetic term, the superpotential term, the Fayet-Iliopoulos term and the field strength for the gauge bosons. Write out the Lagrangian in compontents and find the equations of motion for the auxiliary fields D and F.
Hint: Use the results of Exercise Sheet 8, Exercise 2. The only differences are the Fayet-Iliopoulos contribution and the neutral field Φ_0, which does not couple to the gauge fields.

e) Show that supersymmetry remains unbroken and calculate the mass term for the gauge fields.
Note that the masses of the chiral states stay the same before and after imposing gauge invariance.