Quantum Field Theory III

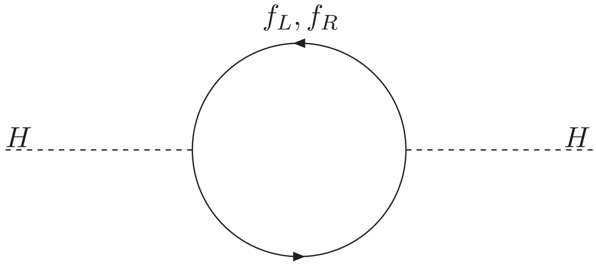
HS 10, Exercise sheet 12

Due date: 15.12.2010

Exercise 1:

The goal of this exercise is to see explicitely how supersymmetry improves the UV behaviour of a quantum field theory. We will look at the radiative corrections to the Higgs mass coming from fermionic and scalar loops.

a) The Yukawa Lagrangian $\mathcal{L} = -\frac{y}{\sqrt{2}} H \bar{f}_L f_R + h.c.$ induces fermionic corrections to the Higgs mass through the following Feynman diagram.

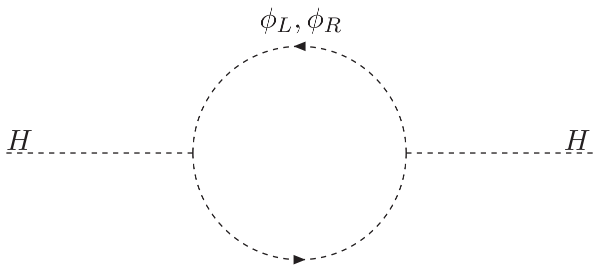


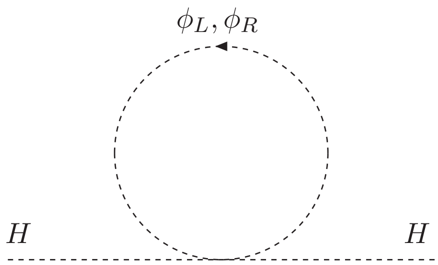
Compute the above Feynman diagram, by using a cut-off regularization. We are only interested in the divergent parts.

b) The Higgs can also interact with scalar particles through the following Lagrangian

$$\mathcal{L} = -\frac{\lambda}{2} H^2 (|\phi_L|^2 + |\phi_R|^2) - H(\mu_L |\phi_L|^2 + \mu_R |\phi_R|^2) - m_L^2 |\phi_L|^2 - m_R^2 |\phi_R|^2.$$

Compute the divergent part of the following two Feynman diagrams contributing to Higgs mass correction using cut-off regularization.





c) What relations among the couplings (λ, y, μ_L, μ_R) and the masses (m_f, m_L, m_R) do we need to cancel the quadratic divergencies? What about logarithmic divergencies?

d) Are these relations fullfilled in supersymmetric theories? What happens to these relations if supersymmetry is broken explicitely by soft-breaking terms, i.e. mass terms and terms with couplings of positive mass dimension?

Exercise 2:
When extending the Standard Model (SM) supersymmetrically, the following terms arise in the superpotential[1]

$$\begin{aligned} W_{\Delta L=1} &= \frac{1}{2}\lambda^{ijk} L_i L_j \bar{e}_k + \lambda'^{ijk} L_i Q_j \bar{d}_k + \mu'^{i} L_i H_u, \\ W_{\Delta B=1} &= \frac{1}{2}\lambda''^{ijk} \bar{u}_i \bar{d}_j \bar{d}_k. \end{aligned}$$

a) Assume that usual baryon number is conserved (values $+1/3$ for Q_i, $-1/3$ for $\bar{u}_i$ and $\bar{d}_i$, 0 for all others). Show that the $W_{\Delta B=1}$ term is forbidden.

b) Find an assignment of lepton numbers, which coincides with the usual one for the SM particles and is conserved by the $W_{\Delta L=1}$ superpotential.

   Hint: The lepton number assignment can be different for the various components in a superfield. That is it can come from an R-symmetry which does not commute with supersymmetry.

[1] The superpotential has to be a gauge-invariant, holomorphic and renormalizable function of chiral superfields.