Particle Physics Phenomenology I

HS 10, Series 2

Due date: 08.10.2010, 1 pm

Exercise 1 and 2 are mandatory. You can choose to do either exercise 3 or 4.

Exercise 1 Calculate the following for a $2 \to 2$ scattering process $(1 + 2 \to 3 + 4)$

(i) the energy E_i^* of the particles and their momenta $|\vec{p}|$ and $|\vec{p}'|$. Determine the asymptotic behaviour of these quantities for $s \gg m_i^2$.

(ii) Show that the scattering angle Θ^* (between particles 1 and 3) is given by

$$\cos \Theta^* = \frac{s(t-u) + (m_1^2 - m_2^2)(m_3^2 - m_4^2)}{\sqrt{\lambda(s, m_1^2, m_2^2)}\sqrt{\lambda(s, m_3^2, m_4^2)}}$$

where

$$\lambda(s, m_1^2, m_2^2) = s^2 + m_1^4 + m_2^4 - 2s \cdot m_1^2 - 2s \cdot m_2^2 - 2m_1^2 m_2^2 = (s - m_1^2 - m_2^2)^2 - 4m_1^2 m_2^2$$

(show also that s, t and u are not independent because they satisfy $s+t+u = \sum_i m_i^2$).

(iii) Insert the extremal values $\cos \Theta^* = \pm 1$ and express u in s, t and the m_i to determine $t_{\min}$ and $t_{\max}$ as functions of s and the m_i, then compute the asymptotic ($s \gg m_i$) behaviour of $t_{\min}$ and $t_{\max}$.

Exercise 2 Consider the following process in the rest frame of the laboratory:

$pp \to ppp\bar{p}$; i.e. a proton collides with a proton at rest, afterwards there should be one additional proton and one additional antiproton.

(i) How big is the threshold energy for this process in the center of mass frame, i.e. how big does the total energy of the protons need to be so this process can take place?

(ii) How big is therefore the treshold energy of the incident proton in the lab frame?

Exercise 3 The PEP2 storage ring (BABAR experiment) at the SLAC collides e^- with an energy of 9.0 GeV with e^+ with an energy of 3.1 GeV to produce a $B^0 - \overline{B^0}$ pair ($m_{B^0} = m_{\overline{B^0}}$=5.280 GeV). The B^0 and the $\overline{B^0}$ have a lifetime of $\tau = 1.542 \cdot 10^{-12} s$. How far do the B^0 and the $\overline{B^0}$ move between creation and decay (in the lab frame)? (calculate the velocity of the B^0 in the rest frame of the $B^0 - \overline{B^0}$ pair first)

Exercise 4 Consider an arbitrary scattering process with n external particles, all incoming and massless ($p_i^2 = 0$). Momentum conservation then implies

$$\sum_{i=1}^{n} p_i = 0.$$

Prove that there are $n(n-3)/2$ independent Lorentz invariants. Does the relation also hold for massive particles?
Hints: Define the invariants as $s_{ij} = (p_i + p_j)^2 = 2p_i.p_j$. Why does this $n \times n$ matrix contain all the invariants? How many conditions does momentum conservation imply on the s_{ij}'s?