

Mechanics Fall 2009, Homework 9

November 30, 2009

1. A Hamiltonian system

Let's assume we have a Hamiltonian $H(p_i, x_i)$, $i = 1, \dots, f$ of a system of point masses with f degrees of freedom.

(i) Calculate the Lagrangian L of the system. What variables does L depend on?

(ii) Apply the calculation from part (i) to a relativistic particle with charge e in a constant electrical field \mathbf{E} . In this case,

$$H = c\sqrt{(mc)^2 + p^2} + e\mathbf{E} \cdot \mathbf{x}. \quad (1)$$

Find the equations of motion!

2. Lennard-Jones potential between two molecules

The classical interaction between two inert gas molecules, each of mass m , is given by the potential

$$V(r) = -(2A/r^6) + (B/r^{12})$$

where A and B are constants greater than zero and $r = |r_1 - r_2|$ is the separation between the molecules.

(a) Give the Hamiltonian for the system.

(b) Describe completely the lowest energy classical state(s) of this system.

(c) If the energy is slightly higher than the lowest [from part (b)], what are the possible frequencies of the motion of the system?