Homework 4 - Euler-Lagrange

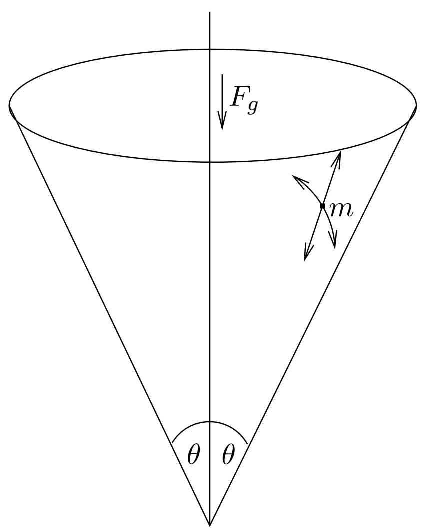
To be handed in: **ETH:** Mon 19-10-09 **UNI:** Wed 21-10-09

1. **Non-Uniqueness of the Lagrangian:** Let L be a Lagrangian for a system of n degrees of freedom. Show by direct substitution that

$$L'(q_1, \ldots, q_n, \dot{q}_1, \ldots, \dot{q}_n, t) = L(q_1, \ldots, q_n, \dot{q}_1, \ldots, \dot{q}_n, t) + \frac{\mathrm{d}}{\mathrm{d}t} F(q_1, \ldots, q_n, t)$$

defines the same set of Euler-Lagrange equations, where F is any three times continuously differentiable function of $q_1, \ldots, q_n$ and t, but not of $\dot{q}_1, \ldots, \dot{q}_n$.

2. **Point Particle glinding in a cone:** Consider a pointlike mass m gliding without friction on the inside of a cone with aperture 2θ. We will assume the gravitational force to be homogeneous and parallel to the axis of the cone.

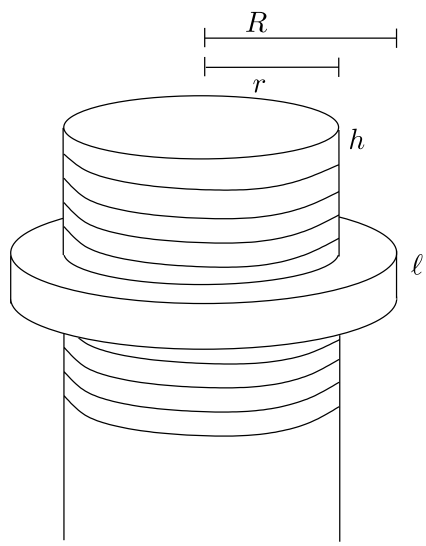


   a) Choose suitable coordinates and determine the Lagrangian.
   b) Compute the Euler-Lagrange equations.

c) Show that the component of angular momentum along the axis of the cone is conserved. Hint: use the Euler-Lagrange equations.

d) Determine the solutions of the Euler-Lagrange equations for which the mass m stays at a constant distance r from the apex. Express r as a function of the angular momentum.

3. **Nut winding down a thread:** Consider a nut winding frictionlessly down the thread of a screw under the influence of a homogeneous gravitational field. The distance covered along the screws axis by one complete rotation of the nut is h. For the purpose of computing the moment of inertia, assume that the nut has a homogeneous density ρ and is a cylinder of length ℓ and radius R with a smaller radius r clinder cut out.



a) Determine the Lagrangian.

b) Compute and solve the Euler-Lagrange equations. How fast does the nut move compared to falling freely.