Sheet IX

`Due: week of 23 November`

Question 1 [*Ricci Tensor*]: Calculate the Ricci tensor R_{ab} for a general static, spherically symmetric spacetime with metric

$$ds^2 = -f(r)dt^2 + h(r)dr^2 + r^2(d\theta^2 + \sin\theta d\phi^2)$$

using the coordinate component method.

Question 2 [*Einstein-Fokker Theory*]: We have seen in the lecture that the Einstein equation reduces to Newtonian gravity in the limit of weak, slowly varying fields. The goal of this exercise is to show that the two requirements general covariance and correct Newtonian limit do not necessarily imply Einstein's equation. One can obtain generally covariant field equations different from those of Einstein if one limits a priori the degrees of freedom of the metric tensor (beyond demanding that the signature be Lorentzian). For example, one might require that the metric is conformally flat, i.e. of the form

$$g_{ab} = \phi^2(x)\,\eta_{ab}\,,$$

where η_{ab} is the flat Minkowski metric.

(i) Show that in this case the Ricci scalar R equals

$$R = -\frac{6}{\phi^3}\eta^{\mu\nu}\partial_\mu\partial_\nu\phi\ . \tag{1}$$

(ii) The analogue of the Einstein field equations is in this case a scalar equation, namely

$$R = 24\pi T\ ,$$

where $T = T_{ab}g^{ab}$ is the trace of the energy-momentum tensor. Show that this reduces, in the weak field quasistationary limit (in which you can ignore time derivatives), to Newtonian gravity

$$\Delta\phi = 4\pi\rho$$
$$\vec{a} = -\vec{\nabla}\phi\,.$$

Hint: Recall that in the linear approximation $g_{ab} = \eta_{ab} + \gamma_{ab}$, and $-\frac{d^2x^\mu}{dt^2} = \Gamma^\mu_{00}$.