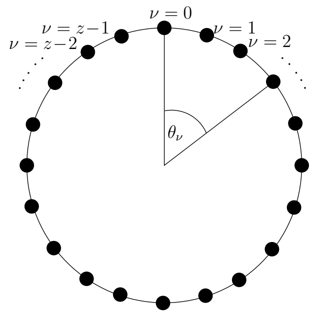
Statistical Physics
Exercise Sheet 3

Exercise 3.1 Statistical Polarization



We study a system of N compasses that can point in z directions with a needle that only hops to the neighboring positions with a rate Γ (see picture).

a) Write the master equation and the H-function (entropy) for this system. What is the equilibrium state of this system?

b) The system is prepared with all the needles pointing equally distributed to the positions $\nu = 0, \ldots, z/3$. What is the H-function of the initial distribution? Compare your result with the equilibrium H-function and interpret the result with respect to the ideal gas entropy.

c) At time $t = 0$, all the needles point in the same direction ($N_0 = N$, $N_\nu = 0$, $\forall \nu \neq 0$). Calculate for long times ($t \gg z^2/\Gamma$) the polarization of the system

$$P(t) = \langle \cos\theta \rangle(t) = \frac{1}{N} \sum_\nu N_\nu(t) \cos\theta_\nu \tag{1}$$

and compare the relaxation of the polarization with the one for the H-function.

d) Starting with the same initial distribution as in b), calculate the exact time dependence of N_ν for the case of $z \to \infty$.
Hint: The N_ν's can be written in terms of Bessel-functions using the Jacobi-Anger expansion

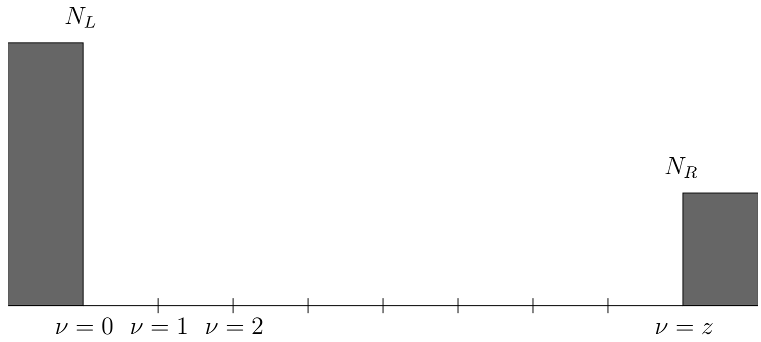
$$e^{iz\cos\theta} = \sum_{n=-\infty}^{\infty} i^n J_n(z) e^{in\theta}. \tag{2}$$

e) Going to a continuum description, $z \to \infty$ and $L = za \to \infty$, while a, the distance between points, vanishes, we find the diffusion equation

$$\dot{\rho}(x,t) = D\partial_x^2 \rho(x,t). \tag{3}$$

What is D in this equation?
Starting with the same initial distribution as in b), calculate the time dependence of the H-function. Interpret your result with respect to the ideal gas.

Exercise 3.2 Particle Current and Entropy Production



We consider a chain with z sites that is connected to two reservoirs such that the boundary conditions $N_0 = N_L$ and $N_z = N_R$ hold for all times (see picture).

a) Find the stationary solution and show that the entropy is not constant in this case. What is the entropy production and where does it come from?

b) We can define a particle current J_ν on the bond between the sites ν and $\nu + 1$ through the continuity equation

$$\dot{N}_\nu = J_{\nu+1} - J_\nu. \tag{4}$$

In the stationary state close to the equilibrium, show that while the current is proportional to the difference in numbers to the left and to the right, the entropy production goes like the square of that difference. What does that mean for the description of transport phenomenon (heat, particles) close to equilibrium?