

Quantum Field Theory I, Exercise Set 10.

HS 08

Due: 04/05 December 2008

1. Reading exercise

Read chapter 13 of the lecture notes carefully.

2. Loop expansion and superficial degree of divergence in φ^4 theory

Consider the φ^4 theory given by the Lagrangian

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \varphi) (\partial^\mu \varphi) - \frac{m^2}{2} \varphi^2 - \frac{\lambda}{4!} \varphi^4 .$$

- (i) Show that, for a given number of external legs, the expansion in the number of loops is equivalent to an expansion in the coupling constant λ .

Hints: Introduce the rescaled field $\varphi = \lambda^{-1/2} \Phi$. What power of λ is carried by each propagator, respectively each vertex? Express the number of loops in terms of the number of internal lines and the number of vertices.

- (ii) Compute the superficial degree of divergence in d dimensions for a diagram in φ^4 theory in terms of number of internal and external legs.
- (iii) Show that there are only two classes of diagrams in φ^4 theory which are potentially divergent in 4 dimensions, corresponding to 2- and 4-point functions. What happens in $d < 4$ dimensions? And in $d > 4$ dimensions?