General relativity, exercise sheet 6.

HS 08 *Due:* Fri, November 7, 2008

1. An ideal fluid

Consider an ideal fluid made of particles of mass m. Suppose that in its rest frame they have isotropically distributed velocities of fixed length v, i.e. $\vec{v} = v\vec{e}$ with $\vec{e}$ uniformly distributed on the unit sphere. Compute the energy density ρc^2 and the pressure p. What happens in the limits

i) $v \to 0$,

ii) $v \to c$ and $m \to 0$, with $mc^2(1 - (v/c)^2)^{-1/2} \to E$ (photons)?

In particular, compare $T^{\mu}{}_{\mu}$ with the trace of the electromagnetic energy-momentum tensor.

2. A variational principle

Consider the variational principle, stated on p. 33, describing the motion $x(\tau)$ of a particle (charge e) falling through an electromagnetic field of potential $A^{\mu}(x)$:

$$\delta \int_{(1)}^{(2)} \mathrm{d}\tau \left(c^2 + \frac{e}{mc}(\dot{x}, A) \right) = 0$$

with fixed endpoints (1) and (2) in space and time. Here $\dot{x} = \mathrm{d}x/\mathrm{d}\tau$ with τ being proper time. Find the corresponding equation of motion.

Hint: The endpoints of τ are not fixed. Thus, parametrize the trajectory by another parameter, which remains fixed under variations.