General relativity, exercise sheet 10.

HS 08 *Due:* Fri, December 5, 2008

1. The magnitude-redshift relation

In Euclidean space the energy flux f of light coming from a source of intensity L (standard candle) at a fixed distance d is

$$f = \frac{L}{4\pi d^2}$$

i) Show that the corresponding relation in a Friedmann universe is

$$f = \frac{L}{4\pi d^2(1+z)^2}\Big(\frac{\chi}{\mathrm{sinn}\chi}\Big)^2 = \frac{L}{4\pi d^2(1+z)^2}\Big(1 - \frac{\Omega_k}{3}(Hd)^2 + \dots\Big)\,, \tag{1}$$

where d is the proper distance, i.e., the present distance between the emitting source (at $\chi = 0$) and us (at χ).
Hints: The energies of the photons relative to sender and receiver are different because of redshift. The proper times between successive photons is too. The area over which the light is spread is $a = 4\pi(R_0 \mathrm{sinn}\chi)^2$, $(a(t_0) = 1)$.

ii) Combine (1) with the distance redshift relation on p.51 of the lecture notes to conclude

$$f = \frac{LH^2}{4\pi z^2}\big(1 - (1-q)z + O(z^2)\big)$$

Remark: the apparent magnitude is essentially $-\log f$.

2. Killing vectors

Find explicit expressions for a complete set of (not necessarily time-like) Killing vector fields for the following spaces:

i) Minkowski space, with metric $ds^2 = dt^2 - dx^2 - dy^2 - dz^2$.
ii) A spacetime with coordinates $\{u, v, x, y\}$ and metric

$$ds^2 = (du\,dv + dv\,du) - a^2(u)dx^2 - b^2(u)dy^2$$

where a and b are unspecified functions of u. This represents a gravitational wave spacetime.
Hint: There are five Killing fields, out of which three are obvious. Two more may be found by the ansatz

$$K^\mu = (0, f, g, h)$$

with f, g, h functions of u (but not of v), and either x or y.