Exercise Sheet IV

Hand in by 29.10.2008

Problem 1 [*Consistency checks on the solution for X^-.*]:

(a) Use

$$\dot{X}^- \pm X^{-\prime} = \frac{1}{\beta\alpha'}\frac{1}{2p^+}\left(\dot{X}^I \pm X^{I\prime}\right)^2 \tag{1}$$

to find $\partial_\tau X^-$ and $\partial_\sigma X^-$. Show that the consistency condition $\partial_\sigma(\partial_\tau X^-) = \partial_\tau(\partial_\sigma X^-)$ holds if the transverse coordinates X^I satisfy the wave equation.

(b) Show that X^-, as calculated in (1), satisfies the wave equation if the transverse coordinates X^I satisfy the wave equation.

(c) Assume that at the open string endpoints some of the transverse light-cone coordinates X^I satisfy Neumann boundary conditions and some satisfy Dirichlet boundary conditions. Prove that X^-, as calculated in (1), will always satisfy Neumann boundary conditions.

Problem 2 [*Rotating open string in the light-cone gauge.*]: Consider string motion defined by $x_0^- = x_0^I = 0$, and the vanishing of all coefficients α_n^I with the exception of

$$\alpha_1^{(2)} = \alpha_{-1}^{(2)*} = a, \qquad\qquad \alpha_1^{(3)} = \alpha_{-1}^{(3)*} = ia.$$

Here a is a dimensionless real constant. We want to construct a solution that represents an open string that is rotating in the (x^2, x^3) plane.

(a) What is the mass (or energy) of this string?

(b) Construct the explicit functions $X^{(2)}(\tau,\sigma)$ and $X^{(3)}(\tau,\sigma)$. What is the length of the string in terms of a and α'?

(c) Calculate the $L_n^\perp$ modes for all n. Use your result to construct $X^-(\tau,\sigma)$. Your answer should be σ-independent!

(d) Determine the value of p^+ using the condition that for this string $X^1(\tau,\sigma) = 0$. Find the relation between t and τ.

(e) Confirm that in your solution the energy of the string and its angular frequency of rotation are related to its length as in the equation

$$\ell = \frac{2c}{\omega} = \frac{2}{\pi}\frac{E}{T_0}.$$