Ferromagnetic metals

Stoner model

$$\mathcal{H} = \sum_{\vec{k},s} \epsilon_{\vec{k}}\, \hat{c}^{\dagger}_{\vec{k}\,s}\, \hat{c}_{\vec{k}\,s} + U \int d^3r \;\; \hat{\rho}_{\uparrow}(\vec{r})\, \hat{\rho}_{\downarrow}(\vec{r})$$

kinetic energy

contact interaction
note: exchange hole

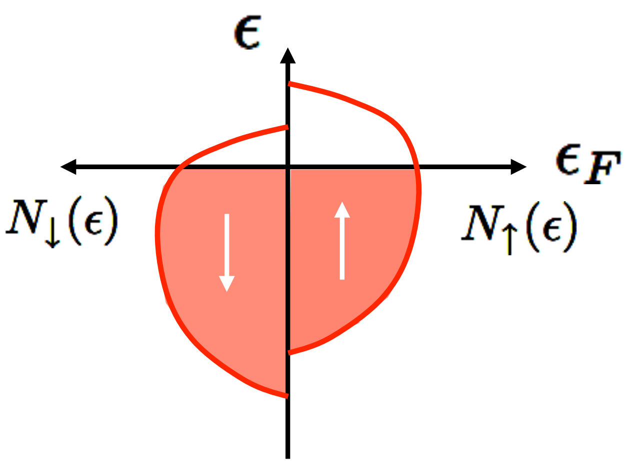
kinetic energy
versus
exchange energy

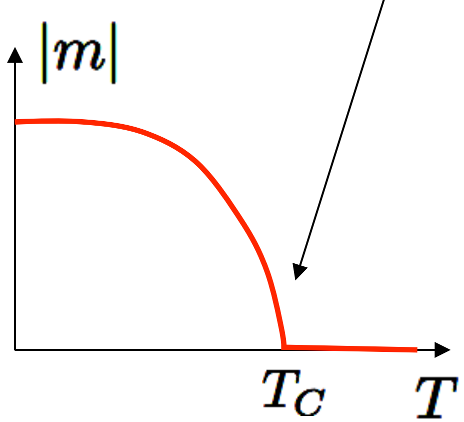
mean field treatment $\hat{\rho}_s(\vec{r}) = n_s + [\hat{\rho}_s(\vec{r}) - n_s]$

mean density of electron with spin *s*

spin polarization $m = n_{\uparrow} - n_{\downarrow} \propto |T - T_C|^{1/2}$

mean field exponent





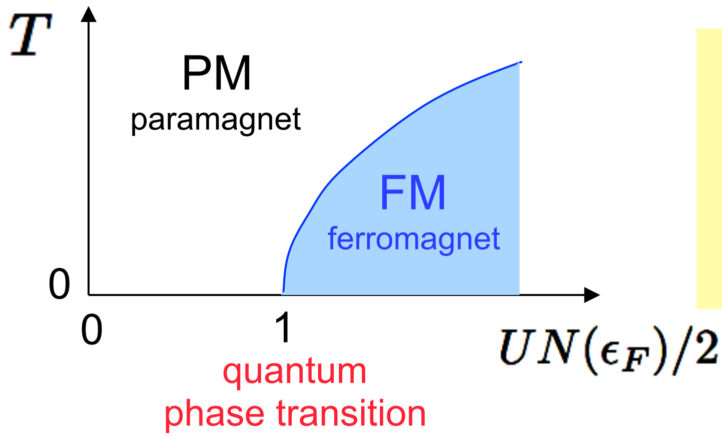
spontaneously broken symmetries

O(3) rotation

$\mathscr{K}$ time reversal

Ferromagnetic metals

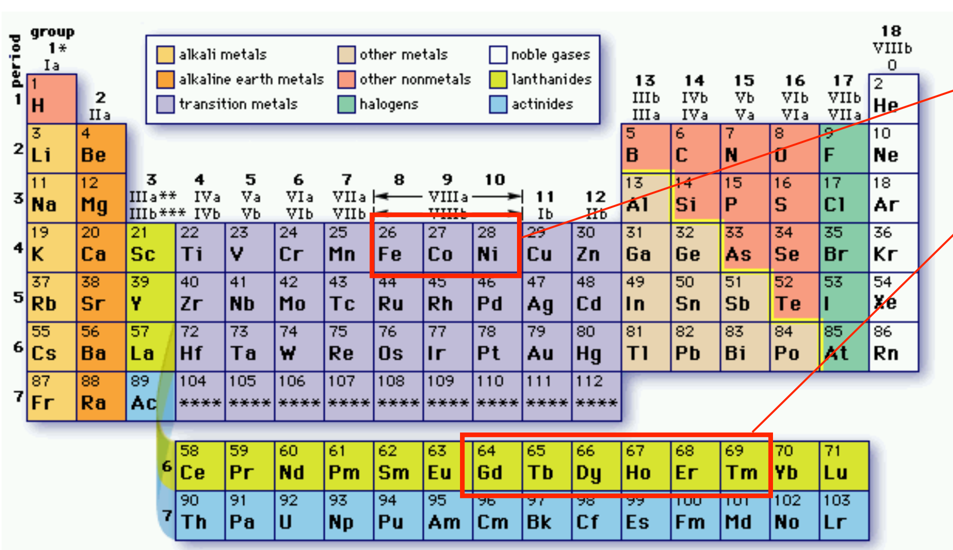
$$k_B T_C \propto \sqrt{1 - \frac{U_c}{U}} \qquad U_c N(\epsilon_F) = 2 \qquad N(\epsilon) = 2\sum_{\vec{k}} \delta(\epsilon - \epsilon_{\vec{k}})$$



Stoner criterion for FM

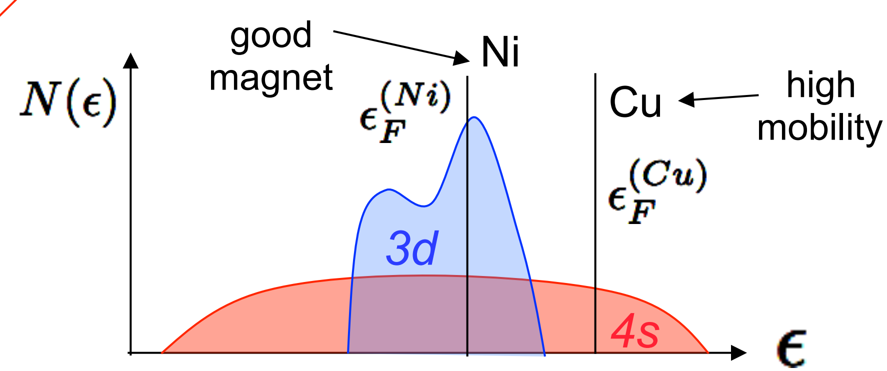
$$U N(\epsilon_F) > 2$$

large $\dfrac{\text{exchange energy}}{\text{kinetic energy}}$



3d-transition metals Fe Co Ni

4f-rare earths Gd Tb Dy Ho Er Tm



Generalized spin susceptibility

Hamiltonian: system in spatial/time dependent magnetic field

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\mathrm{int}} + \mathcal{H}_Z$$

$$\mathcal{H}_0 = \sum_{\vec{k},s} \epsilon_{\vec{k}}\, \widehat{c}^{\dagger}_{\vec{k}\,s}\, \widehat{c}_{\vec{k}\,s} \qquad \mathcal{H}_{\mathrm{int}} = U \int d^3r\, \widehat{\rho}_{\uparrow}(\vec{r})\, \widehat{\rho}_{\downarrow}(\vec{r})$$

$$\mathcal{H}_Z = -\frac{g\mu_B}{\hbar} \int d^3r\, \vec{H}(\vec{r},t) \cdot \widehat{\vec{S}}(\vec{r})$$

$$\widehat{\vec{S}}(\vec{r}) = \frac{\hbar}{2} \sum_{s,s'} \widehat{\Psi}^{\dagger}_{s}(\vec{r})\, \vec{\sigma}_{ss'}\, \widehat{\Psi}_{s'}(\vec{r}) = \frac{\hbar}{2} \begin{pmatrix} \widehat{\Psi}^{\dagger}_{\uparrow}(\vec{r})\, \widehat{\Psi}_{\downarrow}(\vec{r}) + \widehat{\Psi}^{\dagger}_{\downarrow}(\vec{r})\, \widehat{\Psi}_{\uparrow}(\vec{r}) \\ -i\, \widehat{\Psi}^{\dagger}_{\uparrow}(\vec{r})\, \widehat{\Psi}_{\downarrow}(\vec{r}) + i\, \widehat{\Psi}^{\dagger}_{\downarrow}(\vec{r})\, \widehat{\Psi}_{\uparrow}(\vec{r}) \\ \widehat{\Psi}^{\dagger}_{\uparrow}(\vec{r})\, \widehat{\Psi}_{\uparrow}(\vec{r}) - \widehat{\Psi}^{\dagger}_{\downarrow}(\vec{r})\, \widehat{\Psi}_{\downarrow}(\vec{r}) \end{pmatrix}$$

Generalized spin susceptibility

$$\mathcal{H}_Z = -\frac{g\mu_B}{\hbar}\int d^3r\, \vec{H}(\vec{r},t)\cdot \widehat{\vec{S}}(\vec{r}) \qquad \vec{H} = \frac{1}{2}H^+(\vec{q},\omega)e^{i\vec{q}\cdot\vec{r}-i\omega t}e^{\eta t}\begin{pmatrix}1\\-i\\0\end{pmatrix}$$

in x-y-direction

$$\mathcal{H}_Z = -\frac{g\mu_B}{\hbar\Omega}\sum_{\vec{k}} H^+(\vec{q},\omega)\,\widehat{S}^-_{\vec{k},-\vec{q}}\,e^{-i\omega t+\eta t} + \text{h.c.}$$

with $\widehat{\vec{S}}_{\vec{q}} = \int d^3r\, \widehat{\vec{S}}(\vec{r})e^{-i\vec{q}\cdot\vec{r}} = \frac{\hbar}{2\Omega}\sum_{\vec{k},s,s'} c^\dagger_{\vec{k},s}\vec{\sigma}_{ss'}c_{\vec{k}+\vec{q},s'} = \frac{1}{\Omega}\sum_{\vec{k}}\widehat{\vec{S}}_{\vec{k},\vec{q}}$

first goal

field-induced magnetization:

$$M^+_{\text{ind}} = \frac{\mu_B}{\hbar}\langle S^+_{\text{ind}}(\vec{q},\omega)\rangle$$

equation of motion (*U=0*):

$$i\hbar\frac{\partial}{\partial t}\widehat{S}^+_{\vec{k},\vec{q}} = [\widehat{S}^+_{\vec{k},\vec{q}}, \mathcal{H}_0 + \mathcal{H}_Z]$$

Generalized spin susceptibility

$$i\hbar\frac{\partial}{\partial t}\widehat{S}^{+}_{\vec{k},\vec{q}}(t)_{\vec{k},\vec{q}} = (\epsilon_{\vec{k}+\vec{q}}-\epsilon_{\vec{k}})\widehat{S}^{+}_{\vec{k},\vec{q}}(t)+g\hbar\mu_B(\widehat{c}^{\dagger}_{\vec{k}+\vec{q}\uparrow}\widehat{c}_{\vec{k}+\vec{q}\uparrow}-\widehat{c}^{\dagger}_{\vec{k}\downarrow}\widehat{c}_{\vec{k}\downarrow})H^{+}(\vec{q},\omega)e^{-i\omega t}$$

linear order: $\langle S^{+}_{\mathrm{ind}}(\vec{q},t)\rangle = \langle S^{+}_{\mathrm{ind}}(\vec{q},\omega)\rangle e^{-i\omega t+\eta t}$

$$(\epsilon_{\vec{k}+\vec{q}}-\epsilon_{\vec{k}}-\hbar\omega+i\hbar\eta)\langle S^{+}_{\vec{k},\vec{q}}\rangle = -g\hbar\mu_B(n_{\vec{k}+\vec{q}\uparrow}-n_{\vec{k}\downarrow})H^{+}(\vec{q},\omega)$$

$$\langle S^{+}_{\mathrm{ind}}(\vec{q},\omega)\rangle = \frac{1}{\Omega}\sum_{\vec{k}}\langle S^{+}_{\vec{k},\vec{q}}\rangle = \frac{\hbar}{\mu_B}\chi_0(\vec{q},\omega)H^{+}(\vec{q},\omega)$$

$$\chi_0(\vec{q},\omega) = -\frac{g\mu_B^2}{\Omega}\sum_{\vec{k}}\frac{n_{\vec{k}+\vec{q}\uparrow}-n_{\vec{k}\downarrow}}{\epsilon_{\vec{k}+\vec{q}}-\epsilon_{\vec{k}}-\hbar\omega+i\hbar\eta}$$

analog to
Lindhard function

Generalized spin susceptibility

feedback effect through interaction:

$$\mathcal{H}_{\rm int} = \frac{U}{\Omega} \sum_{\vec{k},\vec{k}',\vec{q}'} \widehat{c}^{\dagger}_{\vec{k}+\vec{q}'\uparrow} \widehat{c}_{\vec{k}\uparrow} \widehat{c}^{\dagger}_{\vec{k}'-\vec{q}'\downarrow} \widehat{c}_{\vec{k}'\downarrow} = -\frac{U}{\Omega\hbar^2} \sum_{\vec{q}'} \widehat{S}^{+}_{\vec{q}'} \widehat{S}^{-}_{-\vec{q}'} + const.$$

approximation

$$-\frac{U}{\hbar^2} \langle S^+(\vec{q},\omega)\rangle \widehat{S}^{-}_{-\vec{q}} = -\frac{g\mu_B}{\hbar} H^{+}_{\rm ind}(\vec{q},\omega) \widehat{S}^{-}_{-\vec{q}}$$

$$H^{+}_{\rm ind}(\vec{q},\omega) = \frac{U}{g\mu_B\hbar} \langle S^+(\vec{q},\omega)\rangle$$

$$\begin{aligned} M^+(\vec{q},\omega) &= \frac{\mu_B}{\hbar}\langle S^+(\vec{q},\omega)\rangle \\ &= \chi_0(\vec{q},\omega)[H^+(\vec{q},\omega) + H^{+}_{\rm ind}(\vec{q},\omega)] \\ &= \chi_0(\vec{q},\omega) H^+(\vec{q},\omega) + \chi_0(\vec{q},\omega) \frac{U}{g\mu_B\hbar} \langle S^+(\vec{q},\omega)\rangle \\ &= \chi_0(\vec{q},\omega) H^+(\vec{q},\omega) + \chi_0(\vec{q},\omega) \frac{U}{g\mu_B^2} M^+(\vec{q},\omega) \end{aligned}$$

$$M^+(\vec{q},\omega) = \chi(\vec{q},\omega)H^+(\vec{q},\omega)$$

$$\chi(\vec{q},\omega) = \frac{\chi_0(\vec{q},\omega)}{1-\frac{U}{2\mu_B^2}\chi_0(\vec{q},\omega)}$$

RPA form of spin susceptibility

spin order instabilities: $$1 = \frac{U}{2\mu_B^2}\chi_0(\vec{q},\omega=0)$$

Stoner instability: uniform ferromagnet $\vec{q} \to 0$

$$\chi_0(\vec{q},0) = -\frac{2\mu_B^2}{\Omega}\sum_{\vec{k}}\frac{n_{\vec{k}+\vec{q}\uparrow}-n_{\vec{k}\downarrow}}{\epsilon_{\vec{k}+\vec{q}}-\epsilon_{\vec{k}}} \xrightarrow{\vec{q}\to 0} -\frac{2\mu_B^2}{\Omega}\sum_{\vec{k}}\frac{\partial f(\epsilon_{\vec{k}})}{\partial \epsilon_{\vec{k}}} = \chi_0(T)$$

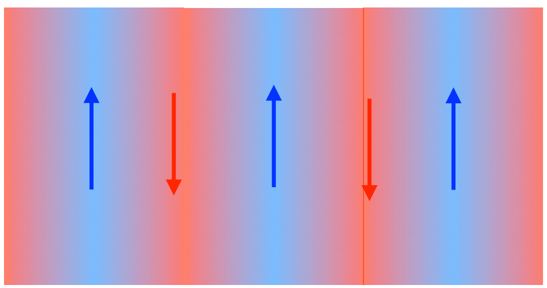
$$1 = \frac{U}{2\mu_B^2}\chi_0 = \frac{UN(\epsilon_F)}{2}\left[1-\frac{\pi^2}{6}(k_BT_C)^2\Lambda_1^2\right]$$

Generalized spin susceptibility

$$1 = \frac{U}{2\mu_B^2}\chi_0(\vec{q}, \omega = 0)$$

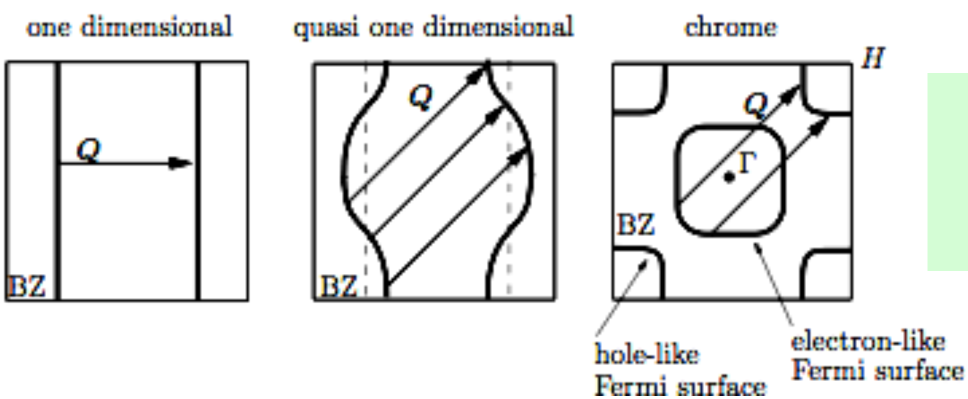
$$\chi(\vec{Q}, 0) = \max_{\vec{q}} \chi_0(\vec{q}, 0)$$

→ spin density wave instability with wavevector $\vec{Q}$



Fermi surface nesting: $\epsilon_{\vec{k}+\vec{Q}} = -\epsilon_{\vec{k}}$

$$\chi_0(\vec{Q}; T) \approx \mu_B^2 N(\epsilon_F) \ln\left(\frac{1.14\epsilon_0}{2k_B T}\right)$$



$$k_B T_N = 1.14\epsilon_0 e^{-2/UN(\epsilon_F)}$$