Exercise 1. ***Drude conductivity and reflectivity of metals and semiconductors***

In exercise 3.1 we have used the semi-classical equations of motion in order to study Bloch oscillations and damping effects. In this exercise we want to derive the Drude conductivity from the semi-classical description and use it to study the reflectivity of metals and semiconductors.

(a) Consider a semi-classical electron subject to an oscillating electric field

$$\boldsymbol{E}(t) = \boldsymbol{E}_0 \mathrm{e}^{-\mathrm{i}\omega t} \tag{1}$$

and model the scattering processes using the relaxation-time approximation with time constant τ. Write the semi-classical equation of motion and solve it in Fourier space to derive the dynamic Drude conductivity

$$\sigma(\omega) = \frac{\omega_p^2}{4\pi} \frac{\tau}{1 - \mathrm{i}\omega\tau} , \tag{2}$$

where the plasma frequency is defined as

$$\omega_p^2 = \frac{4\pi n e^2}{m} . \tag{3}$$

(b) Use the expression for the Drude conductivity to obtain an expression for the reflectivity $R(\omega)$ of a simple metal or semiconductor using the connection between $\sigma(\omega)$ and $R(\omega)$ given in Sec. 6.2.2. of the lecture notes. To take into account the effect of the bound (or core) electrons, use as a phenomenological ansatz for the dielectric function

$$\epsilon(\omega) = \epsilon_\infty + \epsilon_{\mathrm{Drude}}(\omega) - 1 . \tag{4}$$

Here ϵ_∞ is assumed to be constant in the frequency range of interest, related to the fact that the energy scale for exciting core electrons is much higher than the typical energy scales for the itinerant electrons. Plot the reflectivity for the cases $\epsilon_\infty \in \{1, 20\}$ and $\tau\omega_p \in \{2, 40, \infty\}$!

Usually, ϵ_∞ is much larger in semiconductors than in metals. Can you think about a possible explanation for this behavior?

(c) In semiconductors, most electrons are not free to move through the crystal but are bound to the ions. In order to find a better description of their behavior than just using ϵ_∞, model this binding by an additional harmonic potential of the form $V = \frac{1}{2} m \omega_0^2 \boldsymbol{r}^2$ and perform the same analysis as in part (a). How does this influence the reflectivity discussed in part (b)?