Exercise 1. ***Massive gauge boson propagator***

Find the inverse of the operator:

$$\left[\left(\partial^2 + M^2\right) g_{\mu\nu} - \left(1 - \frac{1}{\xi}\right) \partial_\mu \partial_\nu .\right]$$

This corresponds to the propagator of massive vector bosons such as the $W^\pm$ or Z^0.

Exercise 2. ***Axial gauge***

Find the gauge boson propagator in an axial gauge:

$$G^a(A) = n_\mu A^{a\mu},$$

where n is a light-like vector $n^2 = 0$.

Hint. Proceed analogously to the derivation of the bosonic propagator by Fourier transforming and choosing a suitable ansatz for the momentum space propagator.

Exercise 3. ***Path Integral for gauge-fixed QCD***

Given that the gauge-fixed QCD Lagrangian is given by:

$$\mathcal{L} = \mathcal{L}_{YM} + \mathcal{L}_{fermion} + \mathcal{L}_{gauge-fixing} + \mathcal{L}_{ghost}$$

where in the Lorenz gauge:

$$\mathcal{L}_{gauge-fixing} = -\frac{1}{2\xi}\left(\partial^\mu A^a_\mu\right)^2 \qquad \mathcal{L}_{ghost} = (\partial^\mu \bar{\eta}^a) D^{ab}_\mu \eta^b$$
$$\mathcal{L}_{YM} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} \qquad \mathcal{L}_{fermion} = \bar{\psi}^i (i\gamma^\mu D^{ij}_\mu - m\delta^{ij})\psi^j$$

and the generating functional of the full theory is

$$Z = \exp\left\{ i \int d^4 z \mathcal{L}_{int}\left(-i\frac{\delta}{\delta J_A(z)}, i\frac{\delta}{\delta J_\psi(z)}, -i\frac{\delta}{\delta J_{\bar{\psi}}(z)}, i\frac{\delta}{\delta J_\eta(z)}, -i\frac{\delta}{\delta J_{\bar{\eta}}(z)} \right) \right\} Z_0\left[J_\psi, J_{\bar{\psi}}, J_\eta, J_{\bar{\eta}}, J_A\right]$$

(a) Determine all of the possible interaction vertices in QCD.

(b) Given these possible vertices, write down all of the connected diagrams which contribute to the gluon propagator up to $\mathcal{O}(g^2)$.

(c) Write down all of the connected diagrams which contribute to the ghost and fermion propagators up to $\mathcal{O}(g^2)$.