Exercise 1. ***Yukawa Theory from Path Integral Quantization***

Consider the Yukawa theory, where the interaction between the scalar and the fermions has the form $\lambda\,\bar{\psi}\psi\,\phi$.

(a) Calculate the fermionic and scalar propagators through $O(\lambda^2)$.

(b) Derive the Feynman diagrams contributing to the scattering of four fermions at $O(\lambda^2)$.

(c) Derive the Feynman diagrams contributing to the scattering of two fermions and two scalars at $O(\lambda^2)$.

Exercise 2. ***Abelian gauge theories***

Consider the QED Lagrangian

$$\mathcal{L} = \bar{\psi}\,(i\not{D} - m)\,\psi - \frac{1}{4}\,F^{\mu\nu}F_{\mu\nu}$$

where $D_\mu = \partial_\mu - igA_\mu$ is the covariant derivative.

Find the Noether current and the conserved charge corresponding to the invariance under the $U(1)$ global transformation.

Exercise 3. ***Structure constants of*** $SU(N)$

Elements of $SU(N)$ can be represented by $U(\theta) = e^{ig\theta_a T^a}$, where T^a's are the generators of the group.

(a) Show that the generators form a Lie Algebra, i.e.

$$\left[T^a,\, T^b\right] = i\,f^{abc}\,T^c$$

Hint. Consider two independent group elements of $SU(N)$, U, U' and compute the product $U^{'\,-1}U^{-1}U'U$.

(b) Show that the structure constants f^{abc} are real.