Sixth Exercise Sheet due to 10. April

Exercise 1 (Radiative Decoherence) *A harmonic oscillator coupled to a heat bath is described by a Lindblad equation*

$$\dot{\rho} = \mathcal{L}\rho = -i[H,\rho] + \frac{\gamma}{2}(n+1)\left(2a\rho a^* - a^*a\rho - \rho a^*a\right) + \frac{\gamma}{2}(n)\left(2a^*\rho a - aa^*\rho - \rho aa^*\right),$$

where γ is the relaxation rate and n is the mean number of quanta in the thermal equilibrium

$$n = \frac{1}{\exp(\hbar\omega\beta) - 1}.$$

The Hamiltonian $H = \omega(a^*a + 1/2)$ is unimportant for this exercise and can be eliminated from the equation by going to the interaction picture.

Find the stationary state of the equation, i.e. a state σ such that $\mathcal{L}\sigma = 0$. The mean number n appearing in the Lindblad Eq. is a parameter of the model, check that its relation to the temperature is consistent with the stationary state σ.

Let $N(t) := \mathrm{Tr}(a^*a\rho(t))$ be the number of quanta. Show that it is given by

$$N(t) = \exp(-\gamma t)N(0) + n(1 - \exp(-\gamma t)).$$

This show that γ is indeed the relaxation rate.

The equation can be applied for example to a field in the cavity (in that case a^* is the creational operator of a mode of the cavity) or to the motional degree of freedom of a trapped ion (in that case a^* corresponds to the center of mass mode (COM)).