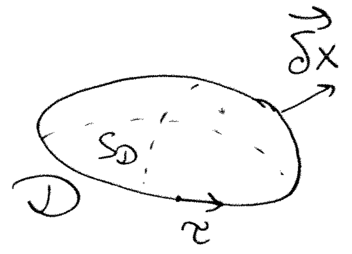
Dislocation motion



S_D - surface where displacement jumps $\left.\vec{u}_+ - \vec{u}_-\right|_{S_D} = \vec{b}$

Dislocation motion produces the change of S_D. With displacement $\vec{\delta x}$, the change of the surface

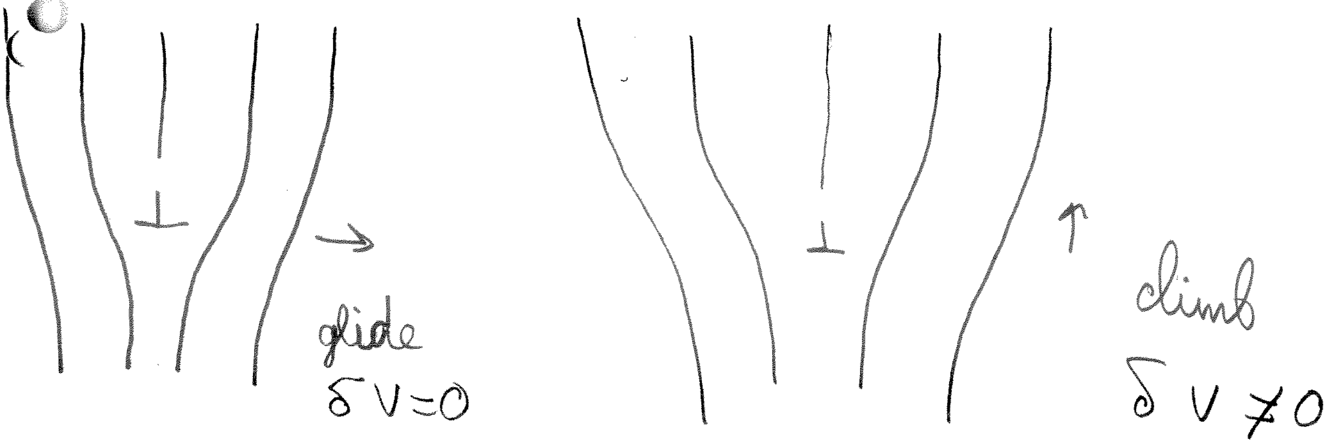
$$\delta S = [\vec{\delta x} \times \vec{dl}] = [\vec{\delta x} \times \vec{\tau}]\, dl$$

The change of volume of the media is

$$\delta V = \vec{b}\, \delta \vec{S} = \vec{\delta x} \cdot [\vec{\tau} \times \vec{b}]\, dl$$

Two different situations:

Volume is unchanged if the motion is in the glide plane $\parallel \tau, b$



glide is easy motion

climb - very hard due to diffusion of the point defects

Forces acting on dislocation

On the surface S_D $\vec{u}_+ - \vec{u}_- = \vec{b}$

Thus w_{ik} has there singularity

$$w_{ik}^{(s.)} = n_i b_k \delta(\xi)$$

$\vec{n}$ is normal to the surface $\vec{\xi} \parallel \vec{n}$

$$u_{ik}^{(s)} = \frac{1}{2}(n_i b_k + n_k b_i)\,\delta(\xi)$$

Since due to dislocation motion S_D is changing then by moving dislocation by $\delta\vec{r}$

$$\delta u_{ik}^{(pl)} = \frac{1}{2}\left\{ b_i [\delta\vec{r} \times \vec{\tau}]_k + b_k [\delta\vec{r} \times \vec{\tau}]_i \right\} \delta^2(r - r_d) \qquad (3)$$

This is <u>plastic deformation</u>

Related with this deformation work due to an external source is

$$\delta R = \int \sigma_{ik}^{ext}\, \delta u_{ik}\, dV$$

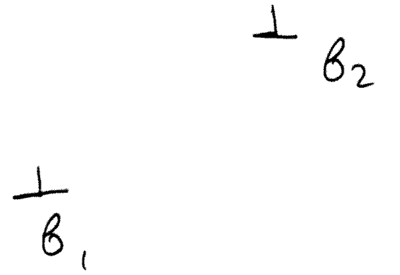
Substituting Eq. (3) we obtain

$$\delta R = \oint \sigma_{ik}^{ext}\, e_{ilm}\, \delta r_l\, \tau_m b_k\, dl$$

Thus we obtain Peach Köhler force

$$\underline{f_i = e_{ikl}\, \tau_k\, \sigma_{lm}\, b_m}$$

Interaction of two edge dislocations



$\tau_z = -1 \quad b_x = b$

$$f_i = e_{ikl}\,\tau_k\,\sigma_{lm}\,b_m \Rightarrow$$

$$f_x = b\,\sigma_{xy}\,, \quad f_y = -b\,\sigma_{xx}$$

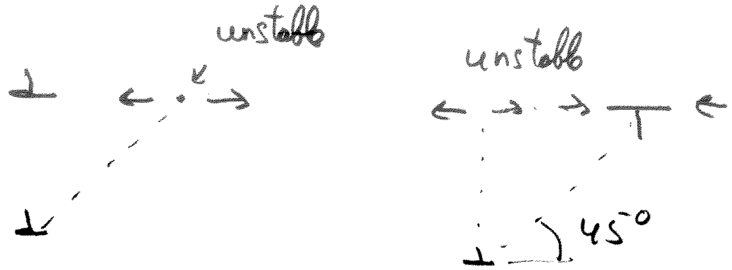
Using expressions for the stress around the edge dislocation we obtain

$$f_x = b_1 b_2 B \,\frac{x(x^2-y^2)}{r^4}$$

$$f_y = b_1 b_2 B \,\frac{y(3x^2+y^2)}{r^4} \qquad B = \frac{\mu}{2\pi(1-\sigma)}$$

$$f_r = \frac{b_1 b_2 B}{r}\,, \quad f_\varphi = \frac{b_1 b_2 B}{r}\sin 2\varphi$$

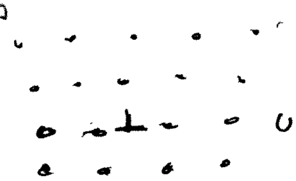
Stable position



In the same gliding plane two opposite dislocations attract each other

Peierls - Nabarro force

In continuum approximation dislocation can glide freely in the glide plane.
But due to the discreteness of the atomic structure there is a finite barrier (Peierls-Nabarro barrier)
To calculate it let us take the discrete version of the energy of the edge dislocation



$$E = \frac{\mu b^2}{4\pi^2(1-\nu)} \sum_{nm} \frac{y_m^2}{\left((x-x_n)^2+y_m^2\right)^2}$$

$$x_n = nb, \quad y_m = mb + \frac{b}{2}$$

We can rewrite it as

$$E = -\frac{\mu b^4}{4\pi^2(1-\nu)} \sum_m y_m^2 \frac{\partial}{\partial y_m^2} \sum_n \frac{1}{(x-nb)^2+y_m^2}$$

The last sum can be calculated using the Poisson formula

$$\sum_{n=-\infty}^{\infty} f(n) = \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} f(x)\, e^{2\pi i k x}\, dx$$

Then

$$\sum_n \frac{1}{(x-nb)^2+y_m^2} = \sum_k \int \frac{e^{i2\pi kt}}{(x-tb)^2+y_m^2}\,dt =$$

$$= \sum_k e^{\frac{i2\pi kx}{b}} \int \frac{e^{i\frac{2\pi k}{b}\xi}}{\xi^2+y_m^2}\,\frac{d\xi}{b} = \quad (\xi = tb-x)$$

$$= \frac{\pi}{b|y_m|} \sum_k e^{i\frac{2\pi kx}{b}}\, e^{-\frac{2\pi k|y_m|}{b}}$$

Then the energy

$$E = -\frac{\mu b^3}{4\pi(1-\sigma)} \sum_m y_m^2 \frac{\partial}{\partial y_m^2} \sum_k \frac{1}{y_m} e^{i\frac{2\pi kx}{b}} e^{-\frac{2\pi k|y_m|}{b}}$$

Keeping only the biggest term with $k=\pm 1$ and smallest $|y_m|$ we obtain

$$E \approx \frac{\mu b^2}{(1-\sigma)} \cos\frac{2\pi x}{b}\, e^{-\frac{2\pi|y_0|}{b}}$$

Force $F = \dfrac{2\pi\mu b}{(1-\sigma)} \sin\dfrac{2\pi x}{b}\, e^{-\frac{2\pi|y_0|}{b}}$

In our case $y_0 = \frac{b}{2}$ and the critical stress $\sigma_m \sim \mu e^{-\pi}$. In "more accurate" model $y_0 \simeq b$