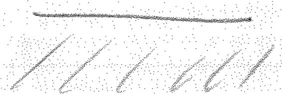
Surface waves (Rayleigh 1885)

There is solution of the wave equations

$$\frac{\partial^2 u}{\partial t^2} - c^2 \Delta u$$

with appropriate $u = u_e, u_t$ and $c = c_e, c_t$ that decays inside the body.

Indeed if body is for $z < 0$



then

$$u = \text{const}\; e^{i(kx-\omega t) + \varkappa z}$$

with $\omega^2 = c^2(k^2 - \varkappa^2) \Rightarrow \varkappa = \left[k^2 - \frac{\omega^2}{c^2}\right]^{1/2}$

In the bulk longitudinal and transverse waves are independent. However, as we observed with reflection of the waves, boundary conditions mix these component and displacement u is some linear combination of u_t and u_e

To determine this combination we should use boundary conditions

$$\sigma_{ik} n_k = 0$$

Because $n \parallel z$ =>

$$\sigma_{xz} = \sigma_{yz} = \sigma_{zz} = 0 \Rightarrow$$

$$u_{xz} = 0 , \quad u_{yz} = 0 , \quad \sigma(u_{xx}+u_{yy})+(1-\sigma)u_{zz}=0$$

Since there is no y dependence =>

$$u_{yz} = 0 \Rightarrow \frac{\partial u_y}{\partial z} = 0$$

But because $u \propto e^{i(kx-\omega t)+\varkappa z}$ then

$$\underline{u_y = 0}$$

Transverse part satisfies

$$\frac{\partial u_{tx}}{\partial x} + \frac{\partial u_{tz}}{\partial z} = 0 \Rightarrow iku_{tx} + \varkappa u_{tz} = 0 \Rightarrow$$

$$u_{tx} = \varkappa_t a \exp(ikx + \varkappa_t z - i\omega t)$$

$$u_{tz} = -ika \exp(ikx + \varkappa_t z - i\omega t)$$

Analogously, for the longitudinal part

$$\frac{\partial u_{ex}}{\partial z} - \frac{\partial u_{ez}}{\partial x} = 0 \Rightarrow i\kappa u_{ez} - \varkappa_e u_{ex} = 0 \Rightarrow$$

$$u_{ex} = \kappa b\, e^{i\kappa x + \varkappa_e z - i\omega t}, \quad u_{ez} = -i\varkappa_e b\, e^{i\kappa x + \varkappa_e z - i\omega t}$$

where a and b are some constants to be determined.

Boundary condition

$\sigma_{xz} \propto u_{xz} = 0$ reads

$$\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} = 0$$

Substituting $u_x = u_{ex} + u_{tx}$, $u_z = u_{ez} + u_{tz}$ with written above $u_{e,t,x,z}$ gives

$$a(\varkappa_t^2 + \kappa^2) + 2b\kappa\varkappa_e = 0 \qquad (1)$$

Equation $\sigma_{zz} = 0$ rewritten through c_t, c_e reads

$$c_e^2 \frac{\partial u_z}{\partial z} + \left(c_e^2 - 2c_t^2\right)\frac{\partial u_x}{\partial x} = 0$$

Substituting $u = u_t + u_t$ and using

$$\frac{\partial u_{tz}}{\partial z} + \frac{\partial u_{tx}}{\partial x} = 0$$ we obtain

$$2 a c_t^2 \varkappa_e k + b\left[c_e^2\left(\varkappa_e^2 - k^2\right) + 2c_t^2 k^2\right] = 0$$

Since $\omega^2 = c_e^2(k^2 - \varkappa_e^2) = c_t^2(k^2 - \varkappa_t^2)$

we can rewrite this boundary condition as

$$2 a \varkappa_t k + b\left(k^2 + \varkappa_t^2\right) = 0$$

This equation is compatible with Eq (1) p. 72 if

$$\left(k^2 + \varkappa_e^2\right)^2 = 4k^2 \varkappa_e \varkappa_t$$

Substituting $\varkappa_{t,t}^2 = k^2 - \frac{\omega^2}{c_{l,t}^2}$ we arrive at

$$\left(2k^2 - \frac{\omega^2}{c_t^2}\right)^4 = 16k^4\left(k^2 - \frac{\omega^2}{c_t^2}\right)\left(k^2 - \frac{\omega^2}{c_l^2}\right)$$

This equation gives dispersion relation

It is clear, that $\omega \propto k$. Let us write it as

$$\omega = c_t k \xi$$

Then we get for $x = \xi^2$

$$\left(1 - \frac{x}{2}\right)^4 = (1 - x)\left(1 - x\frac{c_t^2}{c_l^2}\right)$$

This gives cubic equation for x

$$x^3 - 8x^2 + 8x\left(3 - 2\frac{c_t^2}{c_l^2}\right) - 16\left(1 - \frac{c_t^2}{c_l^2}\right) = 0$$

$x, \xi < 0$ since $\varkappa_t$ is real. $\Rightarrow$

$$c_{surface} = c_t \xi < c_t < c_l$$

In reality $0 < \frac{c_t}{c_l} < \frac{1}{\sqrt{2}} \Rightarrow 0.874 < \xi < 0.955$

From Wikipedia, the free encyclopedia

Rayleigh waves, also known as the **Rayleigh-Lamb Wave** or "ground roll", are a type of surface wave. They are associated on the Earth with earthquakes and subterranean movement of magma, or with any other source of seismic energy, such as an explosion or even a sledgehammer impact, and are also the form of ocean waves.

From a condensed matter physics point of view Rayleigh waves are surface acoustic waves and associated with a huge number of electronic components, the SAW devices, which employ them. SAW devices are mainly used in cellular phones and wireless technology, and their worldwide yearly production is approximated to lie over 1 billion.

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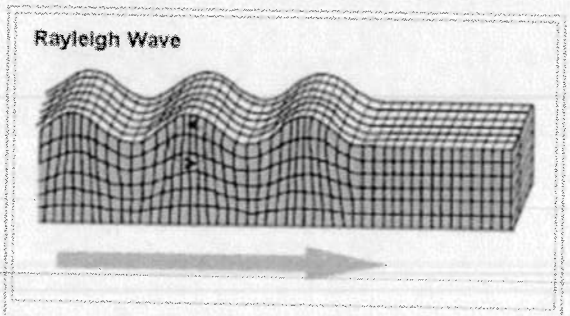
Characteristics

A Rayleigh wave moves across a surface. As it passes, a surface particle moves in a circle or ellipse in the direction of propagation. If one measures particles deeper in the material, the particles move in smaller ellipses, then reach a "no movement" depth, a "node." Below this, harmonics of the Rayleigh wave move particles in regions that alternate elliptical motion with nodes of no-movement. The maximum distance that Rayleigh waves move particles (the amplitude) decreases rapidly with depth in the material.



Picture of a Rayleigh wave.

Since Rayleigh waves are surface waves, the strength, or amplitude, of the waves decreases exponentially with the depth of the earthquake. However, since they are confined to the surface, their amplitude decays only as $\frac{1}{\sqrt{r}}$, where r is the distance the wave has traveled from the earthquake. Surface waves therefore decay more slowly with distance than do body waves, which travel in three dimensions. Large earthquakes may generate Rayleigh waves that travel around the Earth several times before dissipating.

The existence of Rayleigh waves was predicted in 1885 by Lord Rayleigh, for whom they were named. They are distinct from other types of seismic waves, such as P-waves and S-waves, which are both body waves, or Love waves, another type of surface wave. Rayleigh waves are generated by the interaction of P- and S- waves at the surface of the earth. The Rayleigh wave travels with a velocity that is lower than the P-, S-, and Love wave velocities. Emanating outward from the epicenter of an earthquake, Rayleigh waves travel along the surface of the earth at about 10 times the speed of sound in air.

Dispersion

Rayleigh waves in the Earth are also dispersive: Rayleigh waves with a higher frequency travel more slowly than those with a lower frequency. This occurs because a Rayleigh wave of lower frequency has a relatively long wavelength. Long wavelength waves "see" more deeply into the Earth than waves with a short wavelength. Since the speed of waves in the Earth increases with increasing depth, the longer wavelength (low frequency) waves can travel faster than the shorter wavelength (high frequency) waves. Rayleigh waves thus often appear "spread out" on seismograms recorded at distant earthquake recording stations.

Earthquake shaking

Due to their higher speed, the P- and S-waves generated by an earthquake arrive before the surface waves. However, the particle motion of surface waves is larger than that of body waves, so the surface waves tend to cause more damage. In the case of Rayleigh waves, the motion is of a rolling nature, similar to an ocean surface wave. The intensity of Rayleigh wave shaking at a particular location is dependent on several factors:

- The size of the earthquake.
- The distance to the earthquake.

Image:Earthquake wave paths.gif

From Wikipedia, the free encyclopedia

- Image
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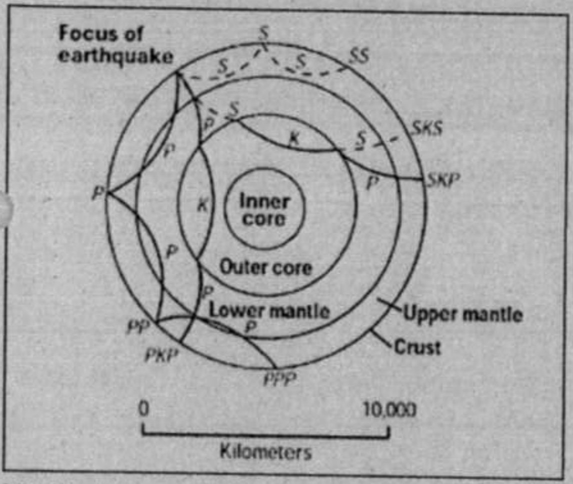
- File links



No higher resolution available.

Earthquake_wave_paths.gif (300 × 250 pixels, file size: 13 KB, MIME type: image/gif)

Cross section of the whole Earth, showing the complexity of paths of earthquake waves. The paths curve because the different rock types found at different depths change the speed at which the waves travel. Solid lines marked P are compressional waves; dashed lines marked S are shear waves. S waves do not travel through the core but may be converted to compressional waves (marked K) on entering the core (PKP, SKS). Waves may be reflected at the surface (PP, PPP, SS).

Seismographs detect the various types of waves. Analysis of such records reveals structures within the Earth.

Source: http://pubs.usgs.gov/gip/interior/fig2.gif ; original upload in english wikipedia, 15 April 2005 by SEWilco

Seismology

From Wikipedia, the free encyclopedia

Seismology (from the Greek *seismos(σεισμός)* = earthquake and *λόγος,logos* = knowledge) is the scientific study of earthquakes and the propagation of elastic waves through the Earth. The field also includes studies of earthquake effects, such as tsunamis as well as diverse seismic sources such as volcanic, tectonic, oceanic, atmospheric, and artificial processes (such as explosions). A related field that uses geology to infer information regarding past earthquakes is paleoseismology. A recording of earth motion as a function of time is called a seismogram.

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Seismic Waves

Earthquakes, and other sources, produce different types of seismic waves which travel through rock, and provide an effective way to image both sources and structures deep within the Earth. There are three basic types of seismic waves in solids: P-waves, S-waves (both body waves) and surface waves. The two basic kinds of surface waves (Rayleigh and Love), can be fundamentally explained in terms of interacting P- and/or S-waves.

Pressure waves (P-waves), are longitudinal waves that travel at maximum velocity within solids and are therefore the first waves to appear on a seismogram.

S-waves, also called Shear waves or secondary waves, are transverse waves that travel more slowly than P-waves and thus appear later than P-waves on a seismogram. Particle motion is perpendicular to the direction of wave propagation. Shear waves do not exist in fluids such as air or water.

Surface waves travel more slowly than P-waves and S-waves, however, because they are guided by the surface of the Earth, and their energy is trapped near the Earth's surface, they can be much larger in amplitude than body waves, and can be the largest signals seen in earthquake seismograms. They are particularly strongly excited when the seismic source is close to the surface of the Earth.

For large enough earthquakes, one can observe the normal modes of the Earth. These modes are excited as discrete frequencies and can be observed for days after the generating event. The first observations were made in the 1960s as the advent of higher fidelity instruments coincided with two of the largest earthquakes of the 20th century - the 1960 Great Chilean earthquake and the 1964 Great Alaskan earthquake. Since then, the normal modes of the Earth have given us some of the strongest constraints on the deep structure of the Earth.

One of the earliest important discoveries (suggested by Richard Dixon Oldham in 1906 and definitively shown by Harold Jeffreys in 1926) was that the outer core of the Earth is liquid. Pressure waves (P-waves) pass through the core. Transverse or shear waves (S-waves) that shake side-to-side require rigid material so they do not pass through the outer core. Thus, the liquid core causes a "shadow" on the side of the planet opposite of the earthquake where no direct S-waves are observed. The reduction in P-wave velocity of the outer core also causes a substantial delay for P waves penetrating the core from the (sesimically faster velocity) mantle.

Seismic waves produced by explosions or vibrating controlled sources are the primary method of underground exploration. Controlled source seismology has been used to map salt domes, faults, anticlines and other geologic traps in petroleum-bearing rocks, geological faults, rock types, and long-buried giant meteor craters. For example, the Chicxulub impactor, which is believed to have killed the dinosaurs, was localized to Central America by analyzing ejecta in the cretaceous boundary, and then physically proven to exist using seismic maps from oil exploration.

Using seismic tomography with earthquake waves, the interior of the Earth has been completely mapped to a resolution of several hundred kilometers. This process has enabled scientists to identify convection cells, mantle plumes and other large-scale features of the inner Earth.