Solid State Theory
Exercise 10

Exercise 10.1 Polarization of a neutral Fermi liquid

Consider a system of neutral spin-1/2 particles each carrying a magnetic moment $\boldsymbol{\mu} = \frac{\mu}{2}\boldsymbol{\sigma}$. An electric field $\boldsymbol{E}$ couples to the particles by the relativistic spin-orbit interaction

$$H_{SO} = \frac{\mu}{2}\left(\frac{\boldsymbol{v}}{c} \times \boldsymbol{E}\right) \cdot \boldsymbol{\sigma}, \tag{1}$$

where $\boldsymbol{\sigma} = (\sigma^x, \sigma^y, \sigma^z)$ is the vector of Pauli spin matrices. In the following we want to calculate the linear response function χ for the uniform polarization

$$\boldsymbol{P} = \chi \boldsymbol{E}. \tag{2}$$

In the presence of the spin-orbit interaction we have to consider a more general situation of a distribution of quasiparticles with variable spin quantization axis. In such a case we must treat the quasiparticle distribution function and the energy as a 2×2 matrix, $(\hat{n}_{\boldsymbol{p}})_{\alpha\beta}$ and $(\hat{\epsilon}_{\boldsymbol{p}})_{\alpha\beta}$, respectively. Furthermore, we require f to be a scalar under spin rotations. In this case f must be of the form

$$\hat{f}_{\alpha\beta,\alpha'\beta'}(\boldsymbol{p},\boldsymbol{p}') = f^s(\boldsymbol{p},\boldsymbol{p}')\delta_{\alpha\beta}\delta_{\alpha'\beta'} + f^a(\boldsymbol{p},\boldsymbol{p}')\boldsymbol{\sigma}_{\alpha\beta} \cdot \boldsymbol{\sigma}_{\alpha'\beta'}. \tag{3}$$

a) Expand $\hat{n}_{\boldsymbol{p}}$, $\hat{\epsilon}_{\boldsymbol{p}}$, and $\hat{f}_{\boldsymbol{\sigma\sigma'}}(\boldsymbol{p},\boldsymbol{p}')$ in terms of the unit matrix $\sigma^0 = \mathbf{1}$ and the Pauli spin matrices $\sigma^1 = \sigma^x$, $\sigma^2 = \sigma^y$, $\sigma^3 = \sigma^z$ and find Landau's energy functional E.

b) Assume that the electric field is directed along the z direction, $\boldsymbol{E} = E_z\hat{z}$. Show that the polarization of such a system is given by

$$P_z = \frac{\partial E}{\partial E_z} = \frac{\mu}{m^* c}\sum_{\boldsymbol{p}}\left(p_y \delta n^1_{\boldsymbol{p}} - p_x \delta n^2_{\boldsymbol{p}}\right). \tag{4}$$

Here, $\delta n^i_{\boldsymbol{p}} = \frac{1}{2}\mathrm{tr}\left[\delta\hat{n}_{\boldsymbol{p}}\,\sigma^i\right]$ and $\delta\hat{n}_{\boldsymbol{p}}$ is the deviation from the equilibrium ($E_z = 0$) distribution function.

c) The application of the electric field changes the quasiparticle energy in linear response according to

$$\delta\tilde{\epsilon}^i_{\boldsymbol{p}} = \delta\epsilon^i_{\boldsymbol{p}} + \frac{2}{V}\sum_{\boldsymbol{p}'} f^{ii}(\boldsymbol{p},\boldsymbol{p}')\delta n^i_{\boldsymbol{p}'} \quad \text{with} \quad \delta n^i_{\boldsymbol{p}} = \frac{\partial n_0}{\partial \epsilon}\delta\tilde{\epsilon}^i_{\boldsymbol{p}} = -\delta(\epsilon^0_{\boldsymbol{p}} - \epsilon_F)\delta\tilde{\epsilon}^i_{\boldsymbol{p}}. \tag{5}$$

Use the ansatz $\delta\tilde{\epsilon}^i_{\boldsymbol{p}} = \alpha\delta\epsilon^i_{\boldsymbol{p}}$ and show that $\alpha = 1/(1 + F^a_1/3)$.

d) Compute χ according to Eq. (2).

Exercise 10.2 Reflectivity of simple metals and Semiconductors

Use the expression for the Drude conductivity,

$$\sigma(\omega) = \frac{\omega_p^2}{4\pi} \frac{\tau}{1 - i\omega\tau}, \tag{6}$$

to obtain an expression for the reflectivity $R(\omega)$ of a simple metal or semiconductor using the connection between $\sigma(\omega)$ and $R(\omega)$ given in Sec. 6.2.2. of the lecture notes. To take into account the effect of the bound (or core) electrons, use as a phenomenological ansatz for the dielectric function

$$\epsilon(\omega) = \epsilon_\infty + \epsilon_{\text{Drude}}(\omega) - 1. \tag{7}$$

Here ϵ_∞ is assumed to be constant in the frequency range of interest, related to the fact that the energy scale for exciting core electrons is much higher than the typical energy scales for the itinerant electrons. Plot the reflectivity for the cases $\epsilon_\infty = 1$ and $\epsilon_\infty = 20$ and $\tau\omega_p = \infty, 40, 2$!

Usually, ϵ_∞ is much larger in semiconductors than in metals. Can you think about a possible explanation for this behavior?

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