Exercise 9.3 Nematic polarisation of a Fermi liquid

We consider a system of electrons under the influence of a general force. This force causes a perturbation of the distribution function, and we are interested in the linear response to the external force. While the quasiparticle energies for the unperturbed system are given by

$$\epsilon_\sigma(\boldsymbol{k}) = \frac{\hbar^2 \boldsymbol{k}^2}{2m^*}, \tag{1}$$

we can take the effects of the external force F into account by

$$\delta\epsilon_\sigma(\boldsymbol{k}) = -\alpha\lambda_\sigma(\boldsymbol{k})F, \tag{2}$$

where α a coupling constant, and λ a dimensionless function describing the angular dependence (anisotropy) of the response. Because a change in the quasiparticle energies will cause a change in the quasiparticle distribution function, which in turn affects feeds back on quasiparticle energy states. Thus, renormalization effects through interactions need to be taken into account. In linear response, the renormalized energy shift is of the same form as the initial shift, Eq. (2), but with a modified coupling constant,

$$\delta\tilde{\epsilon}_\sigma(\boldsymbol{k}) = -\tilde{\alpha}\lambda_\sigma(\boldsymbol{k})F, \tag{3}$$

In this exercise, we discuss the two cases, a spin-independent external force with a form factor of

$$\lambda_\sigma(\boldsymbol{k}) \equiv \lambda_e(\boldsymbol{k}) = \frac{1}{k_F^2}\left(2k_z^2 - k_x^2 - k_y^2\right). \tag{4}$$

defining an ordinary "nematic" deformation of the Fermi surface and a spin-dependent force

$$\lambda_\sigma(\boldsymbol{k}) \equiv \lambda_s(\boldsymbol{k}) = \frac{\sigma}{k_F^2}\left(2k_z^2 - k_x^2 - k_y^2\right). \tag{5}$$

corresponding to "spin nematic" deformation. Note that a nematic deformation can be achieved by uniaxial pressure on a metal. This includes, however, in general also an isotropic deformation.

Hints:
The spherical harmonics are very useful in this exercise.

- You will need the two functions

$$Y_{00} = \frac{1}{\sqrt{4\pi}}, \quad Y_{20} = \sqrt{\frac{5}{16\pi}}\left(3\cos^2\theta - 1\right). \tag{6}$$

- The orthogonality relation is

$$\int d\Omega \; Y_{lm}^*(\theta,\phi) Y_{l'm'}(\theta,\phi) = \delta_{ll'}\delta_{mm'}, \tag{7}$$

  where the integral goes over the whole sphere and $d\Omega = \sin\theta \, d\theta d\phi$.

- Consider the normalized vector $\hat{\boldsymbol{k}}$ and $\hat{\boldsymbol{k}}'$ pointing to (θ, ϕ) and (θ', ϕ') with Θ the angle between them. Then there exists the relation

$$P_l(\cos\Theta) = \frac{4\pi}{2l+1} \sum_{m=-l}^{l} Y_{lm}(\theta, \phi) Y^*_{lm}(\theta', \phi'). \tag{8}$$

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