Exercise 1. *Shor code*

In the lecture we saw how to encode a qubit using Shor's code in order to protect it from arbitrary error on one qubit.

(a) Construct the encoding circuit of the code using Hadamard and CNOT gates.

Let $|\Psi\rangle$ be the nine qubit Shor encoding of the qubit $\alpha|0\rangle + \beta|1\rangle$. Assume that $|\Psi\rangle$ is exposed to a noise process which introduces a bit and a phase flip error on the fourth qubit yielding the faulty state $Z_4 X_x |\Psi\rangle$

(b) What measurements will help you to infer the location and the type of the error (that is, is it a bit flip, a phase flip or both)?

(c) Given the syndrome of the error above, how can you reconstruct the original state $|\Psi\rangle$?

Exercise 2. *Error analysis*

Let $|\Psi\rangle$ be the nine qubit Shor encoding of the qubit $\alpha|0\rangle + \beta|1\rangle$. Assume that the depolarization channel $\mathcal{N}$, which is given by $\mathcal{N}(\rho) = (1-p)\rho + \frac{p}{3}(X\rho X + Y\rho Y + Z\rho Z)$, is acting simultaneously, but independently, on each qubit of $|\Psi\rangle$. Hence, the noise process is formally described by $\mathcal{N}^{\otimes 9}$.

(a) What is the probability that an error occurs that cannot be corrected by the Shor code? Neglect higher order terms in the calculation, i.e. do not take into account when three or more errors occur simultaneously on different qubits.

Hint. Can we fix errors in two qubits in some special cases?

(b) How large can p be such that the concatenation of the Shor code still reduces the error probability?