Non-Abelian Gauge Theory

Outline

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Abstract

Particles and their interactions are described by quantum field theories. Famous examples are QED, QCD and the Standard Model. All of these models are gauge theories, i.e. QFTs with local symmetries. In this proseminar, we will discuss fundamental aspects of gauge theories, e.g. Lie groups, quantisation, renormalisation, anomalies, instantons. We will also come across a supersymmetric theory with many exciting features.

Organisation

Criteria for passing the module:

- Give a pedagogical presentation demonstrating solid understanding of the material.
- Be present at least 80% of the time.
- Hand in a written report of your talk (around 10 pages, in English, as PDF file).

Each presentation should last around 60 minutes, but not more. It is followed by a general discussion. You are encouraged give a computer presentation.

Each student is assigned to a research assistant at the institute as a tutor for their talk. You should contact your tutor at least six weeks before your talk to discuss logistics. You should keep your tutor updated of your work at least once a week.

One week before your talk you are expected to have a draft of your report as well as a finished set of slides which you have to present to your tutor. The report can be handed in up to two weeks after the talk.

Below you can find a list of all the talks. For each talk there is a brief description of the topic, a list of suggested items to be covered as well as some useful references; for more specific guidance and further references, please ask your tutor.

The talks will be presented on **Monday mornings**, usually **9:45–12:45** (exceptions: **16 Apr**, **7 May**, **14 May**) in **HIT F 32** according to the below schedule.

Didactics Seminar

Prof. Christandl has kindly organised a didactics seminar to give you some inspiration for preparing and improving your presentations. This course is given by Thomas Kropf, SRF, and is scheduled as the first session of the proseminar, **Mon, 20 Feb**, **9:45 – 12:00** (venue at ETH Science City to be announced).

Hints

General remarks for designing your talk:

- Try to give a consistent and interesting presentation of the subject. Sometimes less is more: It is important to get the general message across. You do not need to present all the details and every step of each calculation. Nevertheless you should be prepared to provide further details when asked.
- The list of aims and literature is neither complete nor should you consider all the items as mandatory. Please discuss with your tutor what is a good selection of topics, and what can be left out safely (also with regard to the following talks).
- Think about what the audience will be familiar with and what not. Which points do you have to present in detail? Which ones should you rather just sketch out?

You are also encouraged to coordinate your talk with your neighbours where appropriate:

- Avoid excessive overlap between the talks;
- make references to other talks.
- Do not take away your successor's key points;
- rather prepare the audience for the topics to follow.

Regarding your computer presentation:

- Please bring your own laptop, test it at least 10 minutes before the talk.
- Please do not overload the slides. An even balance between formulae and text usually fits this subject well. Figures are especially helpful to get your message across.
- Be careful about colours, e.g. do not use light colours on a white background. A common mistake is to use plain green which looks fine on a computer screen but is illegible on a projector! Use a darker shade instead, e.g. 60% green.

Contents

A Yang–Mills Theory

The following talks provide the basics of classical and quantum non-abelian gauge theory, also known as Yang–Mills theory.

A1 Classical Yang–Mills Theory

Date: 12 Mar, 9:45 **Student:** Anna-Lena Redmann **Tutor:** Dr. Cristian Vergu

Description: We start by defining classical Yang–Mills theory as a gauge theory.

Goals: Electromagnetism, U(1) gauge symmetry; non-abelian gauge symmetry, covariant derivatives, field strength; Yang–Mills action; matrix-valued fields vs. basis in gauge algebra; coupling of scalars and fermions; relevance of Yang–Mills theories to particle physics.

References: [PS, §15.2] [IZ, §12-1] [Ramond, §6.1, 6.2, 6.3]

A2 Lie Groups

Date: 12 Mar, 11:15 **Student:** Dominik Stark **Tutor:** Dr. Constantin Candu

Description: Yang–Mills theory is a model making heavy use of Lie groups, algebras and their representations. This talk shall give an overview of Lie groups concepts relevant to Yang–Mills theory.

Goals: Lie groups, algebras and representations, structure constants f^{abc}; use unitary group $U(N)$ as main example, mention others: $SO(N)$, $Sp(N)$, exceptional cases; ;specific representations: fundamental, adjoint, (anti)-symmetric products, Young tableaux; Casimir invariant, higher Casimirs, C_{f}, C_{A}; exponential map

References: [Georgi, Sternberg, Cornwell] [PS, §15.4]

A3 Quantum Field Theory

Date: 19 Mar, 9:45 **Student:** Juanfernando Angel **Tutor:** Dr. Achilleas Lazopoulos

Description: Quantisation of Yang–Mills theory relies on the framework of quantum field theory. This talk outlines QFT using the path integral approach.

Goals: action, path integral; momentum space version of action, kinetic terms, propagators, interaction terms; derivation of Feynman rules, use massive ϕ^4 theory as main example;

References: [Ramond, §3.1, 3.2, 3.3, 4.1]

A4 Regularisation and Renormalisation

Date: 19 Mar, 11:15 **Student:** Tim Lappe **Tutor:** Dr. João Pires

Description: Most QFT's are plagued by divergences from loops in Feynman graphs, In sensible QFT's one can absorb the divergences into a proper choice of (infinite/running) coupling constants. This talk is to introduce the concept of regularisation and regularisation.

Goals: loops, UV divergences; regularisation, dimensional regularisation and others (cut-off, point-splitting); renormalisation and couterterms, effective action; beta functions, running coupling, field renormalisation; use massive ϕ^4 theory as main example;

References: [PS, §16.5] [Ramond, §4.2, 4.3, 4.5]

A5 Gauge Fixing

Date: 26 Mar, 9:45 **Student:** Murad Tovmasyan **Tutor:** Dr. Florian Goertz

Description: Gauge theories by definition have a local redundancies, therefore the gauge field propagator cannot be uniquely defined. This talks describes breaking of gauge symmetry to obtain well-defined propagators without ruining the properties of the QFT.

Goals: various gauge fixings, axial, Coulomb, Feynman/Landau; Faddeev–Popov ghosts, ghost loops, one-loop gauge propagator with/without ghosts; BRST transformations; Coulomb gauge and Gribov copies;

References: [PS, §16.1, 16.2, 16.4] [Ramond, §7.2, 7.3] [Kaku, §9.4, 9.5, 12.3] [Bertlmann, §3.5, 3.6]

A6 Yang–Mills Quantisation

Date: 26 Mar, 11:15 **Student:** Jens-Daniel Debus **Tutor:** Dr. Kewang Jin

Description: We now apply the QFT framework to Yang–Mills theory and derive the Feynman diagrams.

Goals: use Faddeev–Popov gauge fixed action, details about gauge fixing in next talk, propagators; Feynman rules, with renormalisable matter; background field quantisation; Ward identities;

References: [PS, §16.1, 16.2, 16.6] [Ramond, §8.1, 8.2, 8.3] [IZ, §12-2]

B Applications

We are now in a position to discuss several interesting applications of quantum Yang–Mills theory.

B1 Asymptotic Freedom

Date: 2 Apr, 9:45 **Student:** Cédric Klinkert **Tutor:** Dr. Cristian Vergu

Description: Gauge theories may or may not have the property of asymptotic freedom which makes them reasonable physical models at sufficiently high energies.

Goals: beta functions, dependence on gauge group and matter context; Landau poles, asymptotic freedom; infrared slavery; asymptotic freedom in the standard model;

References: [PS, §16.5, 16.6, 16.7] [IZ, §12-3] [Ramond, §8.6, 8.8]

B2 Lattice Gauge Theory

Date: 2 Apr, 11:15 **Student:** Tobias Rindlisbacher **Tutor:** Dr. Wolfgang Unger

Description: In the lattice approach, spacetime is discretised and QFT's are reduced to QM's of finitely many degrees of freedom. This offers a way to define gauge theory with rigour and perform numerical approximations. In particular one can derive the quark potential at strong coupling through a rectangular Wilson loop giving a hint at confinement.

Goals: Wilson lines, Wilson loops; lattice QFT, lattice gauge field, plaquette action for YM, lattice matter; rectangular Wilson loop, area law, quark potential, confinement; benefits, challenges, difficulties and achievements of the lattice approach; fermions on the lattice, determinant computation, doubling, quenched approximation;

References: [Kaku, §15] [ZinnJustin, §34.1, 34.3] [PS, §15.3]

B3 Higgs Mechanism

Date: 16 Apr, **9:00** **Student:** Robi Pedersen **Tutor:** Dr. João Pires

Description: The Higgs mechanism is a method to assign masses to interacting vector particles without violating renormalisability or other basic principles of QFT. It involves spontaneous breaking of gauge symmetry.

Goals: motivation W and Z bosons; spontaneous symmetry breaking in ϕ^4, tachyon and alternative vacuum; Goldstone boson; coupling to YM in $U(N)$ $|\phi|^4$, eating of Goldstones, massive vector field; counting of on-shell modes before/after symmetry breaking;

References: [PS, §20.1] [IZ, §12-5] [Kaku, §10.2]

B4 Grand Unification

Date: 16 Apr, **10:30** **Student:** Nicholas Wentzlaff **Tutor:** Dr. Giuliano Panico

Description: A dream of theoretical particle physics is to unite all fundamental forces into one. There are convincing hints towards a grand unified theory (excluding gravity) with gauge group $SU(5)$ or $SO(10)$. Unfortunately, the simplest implementations suffer from protons decaying too fast.

Goals: chiral fermion charges; $SU(5)$ unification, $SU(5) \to SU(3) \times SU(2) \times U(1)$ breaking; anomalies; higgs couplings; right-handed neutrinos, $SO(10)$ unification; proton decay, Weinberg angle;

References: [Kaku, §18] [Georgi, §18, 24] [PS, §22.2]

B5 Supersymmetric Yang–Mills Theory

Date: 23 Apr, 9:45 **Student:** Niklas Beisert **Tutor:** —

Description: Supersymmetry is a symmetry which relates bosons and fermions, and thus forces and matter. Supersymmetry requires specially arranged matter content and particle interactions. The resulting cancellations between bosonic and fermionic modes makes supersymmetric models exceptionally stable and gives them interesting properties.

Goals: Coleman–Mandula theorem, (non-extended) supersymmetry algebra; WZ model, super Yang–Mills, supersymmetry transformations; divergences in supersymmetric theories, stability; avoid superspace;

References: [Ramond, §1.8] [Kaku, §20.1, 20.2, 20.5] [PS, §22.4]

C Topological Aspects

The following couple of talks concern in one way or another curious non-trivial effects related to the global structure of spacetime and the fibre bundle of the gauge field.

C1 Anomalies

Date: 23 Apr, 11:15 **Student:** Bettina Meyer **Tutor:** Dr. Giuliano Panico

Description: Symmetries in QFT's can be broken in various ways. This talk introduces the concept of anomalies which result from symmetries being intrinsically incompatible with quantisation itself. Anomalous breakdown of some symmetries has been observed, e.g. in pion decay. For other symmetries, such as gauge symmetry, an anomaly would be disastrous.

Goals: chiral anomaly, abelian anomaly, non-abelian anomaly; absence of gauge anomalies in the standard model; topological term, theta angle, CP violation; anomaly from variation of 5D/6D terms;

References: [Ramond, §8.9] [PS, §19.2, 19.4] [Bertlmann, §4.2, 4.3, 9.1]

C2 Magnetic Monopoles

Date: 30 Apr, 9:45 **Student:** Matteo Fadel **Tutor:** Dr. Constantin Candu

Description: Electrically charged particles couple as sources to the Maxwell equations. Equivalently, magnetically charged particles can be allowed as sources. This has interesting consequences on quantisation.

Goals: Dirac monopole, formulation of Yang–Mills theory using fibre bundles; charge quantisation; electro-magnetic duality, Montonen–Olive duality, $SL(2,\mathbb{Z})$, strong-weak duality;

References: [Kaku, §16.2] [Bertlmann, §6.4]

C3 Instantons

Date: 30 Apr, 11:15 **Student:** Xinyi Chen **Tutor:** Dr. Wolfgang Unger

Description: Instantons are solitonic solutions to the (euclidean) Yang–Mills equations centred at an instant in spacetime. They have finite action and make exponentially suppressed contributions to the Yang–Mills path integral.

Goals: instanton; self-dual Yang–Mills theory; non-perturbative contribution, radius of convergence of perturbation theory; theta vacuum;

References: [Kaku, §16.5, 16.6] [Ramond, §6.3] [Bertlmann, §6.6]

D AdS/CFT Correspondence

The AdS/CFT correspondence is a remarkable exact duality between gauge theories and string theories. It can be used to predict strong-coupling behaviour of particular gauge theories. In the following talks we shall introduce the best-known duality between $\mathcal{N} = 4$ supersymmetric Yang–Mills theory and IIB superstring theory on the $AdS_5 \times S^5$ background.

D1 Planar Limit

Date: 7 May, **9:00** **Student:** Michael Kech **Tutor:** Dr. Claude Duhr

Description: Yang–Mills theories with $U(N)$ gauge group for sufficiently large N have some resemblance to string theories: 't Hooft observed that the expansion in $1/N$ is an expansion in terms of genus of 2D surfaces, analogous to strings in perturbation theory.

Goals: Feynman rules of $U(N)$ Yang–Mills theories, traces of products of generator T^a; double line notation, fat Feynman graphs; Feynman graphs on Riemann surfaces, Euler characteristic, 't Hooft coupling λ and $1/N$ dependence; planar limit, strict large-N limit; string theory qualitatively, worldsheet, relation to string perturbation theory; simplifications at large N, radius of convergence in λ, non-perturbative contributions $\exp(-1/g^2_{\mathrm{YM}})$;

References: [1, §1, 2, 3] [12, §3]

D2 $\mathcal{N} = 4$ Supersymmetric Yang–Mills Theory

Date: 7 May, **10:20** **Student:** Simon Kast **Tutor:** Dr. Cristian Vergu

Description: This talk introduces the 4D Yang–Mills theory with the maximum amount of supersymmetry. The large amount of supersymmetry puts tight constraints on the model which make it essentially unique. This model turns out to have a host of remarkable features that are all but obvious at first sight.

Goals: fields and action for $\mathcal{N} = 4$ SYM; uniqueness, coupling constants; dimensional reductions from 10D; extended supersymmetry transformations; (super)conformal symmetry, beta function, why finiteness; special properties: finiteness, exact conformal symmetry, Montonen-Olive electromagnetic duality, integrability ...

References: [16] [17, §1.1] [18]

D3 AdS/CFT Correspondence

Date: 7 May, **11:40** **Student:** Praxitelis Ntokos **Tutor:** Dr. Florian Goertz

Description: This talk introduces anti-de Sitter spaces and string theory on the $AdS_5 \times S^5$ background. The holographic duality to $\mathcal{N} = 4$ SYM on the boundary of AdS_5 is explained.

Goals: Green–Schwarz string theory on $AdS_5 \times S^5$, coset space sigma model, action, coupling constants; geometry of AdS, boundary, isometries, holography; formal statement; matching of coupling constants, matching of observables, e.g. Wilson loops vs. strings ending on the boundary; strong-weak problem;

References: [BBS, §12.3] [19] [20, 18, 21, 22, 2, 3]

D4 Conformal Field Theory

Date: 14 May, **9:00** **Student:** Michael Groher **Tutor:** Dr. Kewang Jin

Description: $\mathcal{N} = 4$ SYM is a 4D conformal field theory. This talk is to introduce the objects of interest in a CFT: local operators and their correlation functions.

Goals: conformal symmetry, local gauge-invariant operators, two-point functions, position space representation for field propagators; scaling dimensions, classical dimensions, anomalous dimensions; three-point functions, OPE; AdS/CFT dictionary for local operators;

References: [18] [FMS] [4, §3]

E Integrable Spin Chains in $\mathcal{N} = 4$ SYM

The last part of the proseminar deals with the exciting appearance of a elementary model of magnetism in the spectral problem of a gauge theory.

E1 Dilatation Operator

Date: 14 May, **10:20** **Student:** Frédéric Dreyer **Tutor:** Dr. Marius de Leeuw

Description: Scaling dimensions receive quantum corrections which can be computed in perturbation theory. Alternatively they can be measured as the eigenvalues of the dilatation generator of the conformal algebra. This talk presents the determination of the one-loop dilatation generator.

Goals: sketch of anomalous dimension computation: contributing diagrams, operator renormalisation, logarithmic behaviour and anomalous dimensions; local operators in the $SU(2)$ sector, computation of the dilatation operator; planar and non-planar contributions; construction of the dilatation generator.

References: [17, §2.1.1] [5] [6] [7]

E2 Heisenberg Spin Chain

Date: 14 May, **11:40** **Student:** Nicola Notari **Tutor:** Dr. Andrey Lebedev

Description: A classical model of quantum mechanics and magnetism makes appearance in the dilatation generator for this gauge theory. This talk is to define the Heisenberg (XXX) spin chain model and its spectral problem. A remarkable property of this model is its integrability which leads to the solution presented in the subsequent talks.

Goals: Heisenberg's spin chain model, Hamiltonian, spectral problem; ferromagnetic and antiferromagnetic vacuum; some simple example for the Hamiltonian operator, its matrix representation and diagonalisation on a short spin chain (no Bethe ansatz); integrability, higher conserved Hamiltonians, potentially: R/Lax-matrix, Yang–Baxter-equation;

References: [8] [9] [10, §2, 3] [11, §3]

E3 Coordinate Bethe Ansatz

Date: 21 May, 10:00 **Student:** Marc Biber **Tutor:** Dr. Andrey Lebedev

Description: The Heisenberg spin chain has an exact solution in terms of a simple set of algebra equations, the so-called Bethe equations. In this talk these equations are derived by the coordinate Bethe ansatz that turns the model into a scattering problem of magnons.

Goals: Coordinate Bethe ansatz; vacuum, vacuum energy; one-magnon state, dispersion relation; two-magnon state, scattering phase; many-magnon states, Bethe equations; simple solution of the Bethe equations, comparison to corresponding Hamiltonian eigenvalue.

References: [8] [9] [13, §2]

E4 Algebraic Bethe Ansatz

Date: 21 May, 11:30 **Student:** Luca Papariello **Tutor:** Dr. Marius de Leeuw

Description: A key object for the solution of integrable spin chains is the R-matrix which obeys the Yang–Baxter equation. The R-matrix can be applied to the construction of energy eigenstates. This talk describes how to derive the Bethe equations using the algebra of monodromy matrix elements.

Goals: Lax/R-matrix, Yang-Baxter-equation; transfer matrix and conserved charges; monodromy matrix; elements A, B, C, D, algebraic relations; vacuum, creation and annihilation operators; diagonal elements, undesirable terms, algebraic derivation of the Bethe equations.

References: [11, §3, 4] [10, §3, 4, 5] [14] [17, §4.1] [15, §2.2, 2.3] [13, §3]

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