Solid State Theory
Exercise 9

Exercise 9.1

Compute the integral

$$S(\omega, \boldsymbol{q}) = \frac{1}{\Omega} \sum_{\boldsymbol{k}'} n_{0,\boldsymbol{k}'} \left(1 - n_{0,\boldsymbol{k}'+\boldsymbol{q}}\right) \delta\left(\epsilon_{\boldsymbol{k}'+\boldsymbol{q}} - \epsilon_{\boldsymbol{k}'} - \hbar\omega\right) \tag{1}$$

of Sect. 5.1 in the lecture notes for $\hbar\omega \leqslant \hbar^2/2m(2qk_F - q^2)$.

Exercise 9.2 Uniaxial Compressibility

We consider a system of electrons upon which an uniaxial pressure in z-direction acts. Assume that this pressure causes a deformation of the Fermi surface $k \equiv k_F^0$ of the form

$$k_F(\phi, \theta) = k_F^0 + \gamma \frac{1}{k_F^0}\left[3k_z^2 - (k_F^0)^2\right] = k_F^0 + \gamma k_F^0[3\cos^2\theta - 1], \tag{2}$$

where $\gamma = (P_z - P_0)/P_0$ is the anisotropy of the applied pressure.

a) Show that for small $\gamma \ll 1$, the deformed Fermi surface $k_F(\phi, \theta)$ encloses the same volume as the non-deformed one, k_F^0, where terms of order $\mathcal{O}(\gamma^2)$ can be neglected.

b) The deformation of the Fermi surface effects a change in the distribution function of the electrons. Using Landau's Fermi Liquid theory, calculate the uniaxial compressibility

$$\kappa_u = \frac{1}{V} \frac{\partial^2 E}{\partial P_z^2}, \tag{3}$$

which is caused by the deformation given in Eq. (2) (E denotes the Landau energy functional).

Office hour:
Monday, April 30, 2012 - 15:00 to 16:00
HIT K 11.3
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