NEUTRINOS

Let us start by assuming that neutrinos are massless (nowadays we know that this is not the case, but nevertheless neutrinos have very small masses)

For massless fermions we can use the CHIRAL REPRESENTATION of Dirac gamma matrices

$$\sigma_\mu = (1, \underline{\sigma}) \qquad \gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \bar{\sigma}_\mu & 0 \end{pmatrix} \qquad \gamma_5 = \begin{pmatrix} -1 & \\ & 1 \end{pmatrix}$$

$$\bar{\sigma}_\mu = (1, -\underline{\sigma})$$

DIRAC EQUATION SPLITS INTO TWO INDEPENDENT EQS

$$i\psi_L^\dagger \bar{\sigma}\cdot\partial\psi_L = 0$$

$$i\psi_R^\dagger \sigma\cdot\partial\psi_R = 0$$

$$\psi = \begin{pmatrix} \chi_L \\ \chi_R \end{pmatrix}$$

χ_L, χ_R 2-component WEYL SPINORS

They transform under the $\left(\frac{1}{2}, 0\right)$ and $\left(0, \frac{1}{2}\right)$ representations of the Lorentz group (a Dirac spinor corresponds to the $\left(\frac{1}{2}, 0\right) \oplus \left(0, \frac{1}{2}\right)$ representation)

$$\frac{1-\gamma_5}{2}\psi = \begin{pmatrix} \chi_L \\ 0 \end{pmatrix}$$

We can show that $\chi_L^T \sigma_2 \chi_L$ is a scalar

(since $\sigma_{2\alpha\beta} = -i\,\varepsilon_{\alpha\beta}$ sometimes such terms are written also as $\chi_L \chi_L$)

$\Rightarrow$ SUCH SCALAR TERMS CORRESPOND TO A NEW MASS TERM IN THE LAGRANGIAN: MAJORANA MASS

$$\mathcal{L} = i\chi_L^\dagger \sigma\cdot\partial\chi_L + \frac{im}{2}\left(\chi_L^T \sigma_2 \chi_L - \chi_L^\dagger \sigma_2 \chi_L^*\right)$$

this Lagrangian implies the Klein-Gordon equation $(\Box + m^2)\chi_L = 0$

Alternatively one can define a Majorana 4-component spinor out of χ_L: $\psi_M = \begin{pmatrix} \chi_L \\ -\sigma_2 \chi_L^* \end{pmatrix}$

transforms like a Dirac spinor but is SELFCONJUGATE $\psi_M^c = \psi_M$

Majorana mass terms are forbidden in general if the fermion transforms nontrivially under the gauge group* ($\chi_L \chi_L$ is in general not a singlet!)

But what about ν_R?

ν_R is a singlet under $SU(2)_L \otimes U(1)_Y$ $\Rightarrow$ Either neutrinos are massless and ν_R does not exist or neutrinos are massive

* A fermion in a REAL representation of the gauge group may have a Majorana mass

$$\chi_L^T \sigma_2 \chi_L \rightarrow \chi_L^T U^T \sigma_2 U \chi_L \equiv \chi_L^T \sigma_2 U^\dagger U \chi_L = \chi_L^T \sigma_2 \chi_L$$

if U is orthogonal

example $[T_a, T_b] = i f_{abc} T_c$

$(T^a)_{bc} = i f_{abc}$ $\quad (-T^a)^* = T^a$ $\quad$ the adjoint representation is REAL

In SUSY models at each vector boson (which transforms in the adjoint representation of the gauge group) corresponds a fermion (GAUGINO) in the same representation

$\Rightarrow$ GAUGINOS MAY HAVE MAJORANA MASSES

There is experimental evidence that neutrinos do have mass

$\Rightarrow$ We can add a Dirac mass like for the other fermions $m(\bar{\nu}_L \nu_R + h.c)$ (through Yukawa with Higgs field)

But then the ν_R exists and we cannot exclude the existence of a Majorana mass term for the ν_R $\quad M \nu_R \nu_R$

This mass can be completely unrelated to the electroweak scale and could be very large

See saw mechanism

$$\begin{array}{c|cc} & \nu_L & \nu_R \\ \hline \nu_L & 0 & m \\ \nu_R & m & M \end{array} \qquad \text{i.e.} \quad \begin{pmatrix} 0 & m \\ m & M \end{pmatrix}$$

m : Dirac mass $\Rightarrow$ should be related to the EW scale and so of the order of the other fermion masses

M : Majorana mass for ν_R $\Rightarrow$ it is the only Majorana mass term that is possible in the SM (because ν_R is a singlet) could be very large $M \gg m$

$\downarrow$

If we diagonalize mass matrix we get

2 eigenvalues : $(m \ll M)$

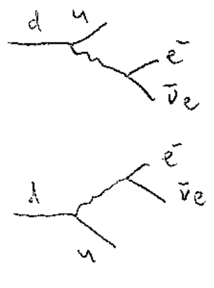
$\frac{m^2}{M}$ and M $\Rightarrow$ the first corresponds to a light, predominantly left handed eigenvector

the second corresponds to a heavy, predominantly right handed eigenvector

$\Rightarrow$ THIS IS THE COMMON EXPLANATION OF THE LIGHTNESS OF NEUTRINO MASSES

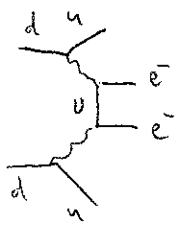
NEUTRINOLESS DOUBLE β DECAY

double beta decay



$A(Z,N) \to A(Z+2, N-2) + 2e^- + 2\bar{\nu}_e$

neutrinoless double beta decay



possible if neutrino is a MAJORANA PARTICLE

Neutrino oscillations

Let us assume that we have only two flavours

$$\begin{pmatrix} \nu_\mu \\ \nu_e \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

ν_1, ν_2 mass eigenstates

$$\nu_1(t) = \nu_1(0)\, e^{-iE_1 t}$$

$$\nu_2(t) = \nu_2(0)\, e^{-iE_2 t}$$

$$\nu_\mu = \cos\theta\, \nu_1 + \sin\theta\, \nu_2$$

$$\nu_e = -\sin\theta\, \nu_1 + \cos\theta\, \nu_2$$

$\nu_e(0) = 0 \Rightarrow$ we start with ν_μ only

$$\nu_\mu(0) = \cos\theta\, \nu_1(0) + \sin\theta\, \nu_2(0)$$

$$0 = -\sin\theta\, \nu_1(0) + \cos\theta\, \nu_2(0) \quad \Rightarrow \quad \nu_1(0) = \frac{\cos\theta}{\sin\theta}\, \nu_2(0)$$

$$\nu_\mu(0) = \cos\theta \frac{\cos\theta}{\sin\theta}\, \nu_2(0) + \sin\theta\, \nu_2(0) = \frac{1}{\sin\theta}\, \nu_2(0) \quad \Rightarrow \quad \begin{aligned} \nu_1(0) &= \cos\theta\, \nu_\mu(0) \\ \nu_2(0) &= \sin\theta\, \nu_\mu(0) \end{aligned}$$

$$\nu_\mu(t) = \cos\theta\, \nu_1(t) + \sin\theta\, \nu_2(t)$$

$$= \cos^2\theta\, \nu_\mu(0)\, e^{-iE_1 t} + \sin^2\theta\, \nu_\mu(0)\, e^{-iE_2 t}$$

$$\Rightarrow \frac{\nu_\mu(t)}{\nu_\mu(0)} = \cos^2\theta\, e^{-iE_1 t} + \sin^2\theta\, e^{-iE_2 t}$$

$$\frac{I_\mu(t)}{I_\mu(0)} = \left| \cos^2\theta + \sin^2\theta\, e^{-i(E_2-E_1)t} \right|^2$$

$$= \left(\cos^2\theta + \sin^2\theta\, e^{-i(E_2-E_1)t}\right)\left(\cos^2\theta + \sin^2\theta\, e^{i(E_2-E_1)t}\right) \qquad \Delta E = E_2 - E_1$$

$$= \cos^4\theta + \sin^4\theta + \cos^2\theta \sin^2\theta \left(e^{i\Delta E t} + e^{-i\Delta E t}\right)$$

$$= \cos^4\theta + \sin^4\theta + 2\cos^2\theta \sin^2\theta \left(\cos \Delta E t - 1 + 1\right)$$

$$= 1 + 2\cos^2\theta \sin^2\theta \left(\cos \Delta E t - 1\right)$$

$$= 1 - 4\cos^2\theta \sin^2\theta \sin^2 \frac{\Delta E t}{2} = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E}$$

$$E_1 = \sqrt{\underline{p}^2 + m_1^2} \approx |\underline{p}| \left(1 + \frac{m_1^2}{2\underline{p}^2}\right) \qquad \Delta E = \frac{\Delta m^2}{2|\underline{p}|}$$

ν_μ "disappearence" driven by Δm^2, L, E

⇒ <u>NOT SENSITIVE TO ABSOLUTE MASS SCALE BUT ONLY TO Δm^2 !</u>

$$P \sim \sin^2\left(\frac{1.27\,\Delta m^2 L}{E}\right)^{*}$$

Δm^2 in eV^2
L in m
E in MeV

Experiments are characterized by E and L ⇒ <u>SENSITIVE TO DIFFERENT Δm^2</u>

<u>Solar neutrinos</u>

electron neutrinos produced in the solar termonuclear reactions

Solar neutrino problem : all experiments show a neutrino flux smaller than the one predicted by the Solar SM

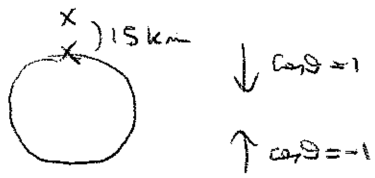
Most popular explanation $\nu_e \to \nu_\mu (\nu_\tau)$

SNO experiment (2001) ⇒ clear evidence of solar neutrino oscillations (non ν_e component greater than zero at 5σ) PRL 89 (2002) 011301

<u>Atmosferic neutrinos</u>

They are produced by the interaction of cosmic rays with earth's atmosphere (at ~15 km)

SuperKamiokande (1998) ⇒ evidence for atmospheric neutrino oscillations (ν_μ disappearence)



Deficit in ν_μ coming up ($\cos\theta = -1$) with respect to those going down ($\cos\theta = 1$)

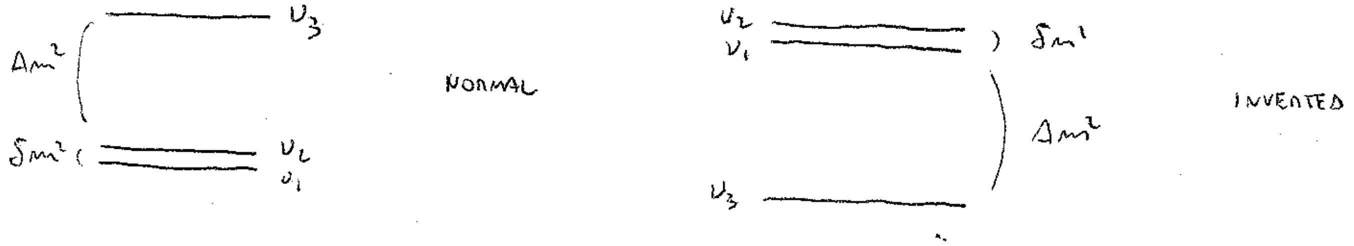
$\delta m^2 = 7.7 \cdot 10^{-5}\ eV^2$ SOLAR

$\Delta m^2 = 2.4 \cdot 10^{-3}\ eV^2$ ATMOSPHERIC

$\delta m^2 / \Delta m^2 \sim \frac{1}{30}$

⇒ VERY DIFFICULT TO BE SENSITIVE TO BOTH MASS SPLITTINGS IN A SINGLE EXPERIMENT

Two possibilities for neutrino mass spectrum



PONTECORVO-MAKI-NAKAGAWA-SAKATA MATRIX (PMNS)

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

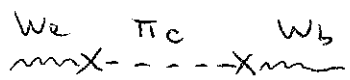
analogous to CKM matrix for quarks

Custodial symmetry

We consider the EW theory spontaneously broken without specifying the Higgs sector.
We want to show that the relation $m_W^2 = m_Z^2 \cos^2\theta$ is a consequence of a symmetry.
Let us consider the gauge currents and the corresponding Goldstone bosons.
The most general parametrization of the matrix element $\langle 0 | J_\mu^a | \pi^b \rangle$ is

$$\langle 0 | J_\mu^a | \pi^b \rangle = i f^{ab} P_\mu$$

The lagrangian will contain a term of the form $g J_\mu^a W_a$ so the mass matrix is generated by the diagram



$$g_a g_b f^{ac} f^{bc}$$

We now assume that there is an unbroken $SU(2)_V$ global symmetry under which the three Goldstone bosons transform as a TRIPLET $\Rightarrow$ one can show that in this case $f^{ab} = f \delta^{ab}$ (Schur's lemma) (the same argument is used for the pions in QCD)

We now impose that the $U(1)_{EM}$ is unbroken

$$\langle 0 | J_\mu^{L3} + \frac{J_\mu^Y}{2} | \pi^3 \rangle = 0 \quad \Rightarrow \quad \langle 0 | J_\mu^Y | \pi^3 \rangle = -i 2 f P_\mu$$

The mass matrix can be written as

$$f^2 \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & Y \end{array} \\ \begin{pmatrix} g^2 & & & \\ & g^2 & & \\ & & g^2 & -gg' \\ & & -gg' & g'^2 \end{pmatrix} \end{array}$$

$$(g^2-\lambda)(g'^2-\lambda) - g^2 g'^2 = 0$$

$$\lambda^2 - \lambda(g^2+g'^2) = 0 \quad \Rightarrow \quad \begin{cases} \lambda = 0 & \text{photon} \\ \lambda = g^2+g'^2 \end{cases}$$

$$\Rightarrow \quad m_W^2 = m_Z^2 \frac{g^2}{g^2+g'^2} \qquad \boxed{\rho = 1}$$

We see that the result $\rho=1$ comes out quite generally, without making other assumptions on the structure of the Higgs sector

Let us now assume that the Higgs sector is the one of the SM

Instead of introducing the Higgs as a complex doublet $\phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_3+i\phi_4\\ \phi_1+i\phi_2\end{pmatrix}$

we can define

$$M = \begin{pmatrix}\phi_1+i\phi_2 & -(\phi_3-i\phi_4)\\ \phi_3+i\phi_4 & \phi_1-i\phi_2\end{pmatrix} \qquad \det M = \phi_1^2+\phi_2^2+\phi_3^2+\phi_4^2 \equiv \phi^2$$

$$\mathcal{L} = \frac{1}{4}\mathrm{Tr}\left(\partial_\mu M^+\partial^\mu M\right) - \frac{\lambda}{4}\left(\det M - v^2\right)^2 \qquad \text{neglect gauge interactions}$$

$SU(2)_L \otimes SU(2)_R$ invariance $\quad M \to g_L^+ M g_R \quad \Rightarrow$ spontaneously broken to $SU(2)_V$

Note also that Yukawa interaction could be written

as

$$\mathcal{L}_Y = K\,(u_L^+\ d_L)\, M \begin{pmatrix}u_R\\ d_R\end{pmatrix} + h.c. \qquad \text{ONLY IF } m_u = m_d\ !$$

$\Rightarrow$ $SU(2)_L \otimes SU(2)_R$ symmetric

The symmetry of the Yukawa interaction is broken already at tree level by $m_u - m_d$ diff.

So, if we neglect gauge interactions the Lagrangian can be written in this $SU(2)_L \otimes SU(2)_R$ symmetric form and the SCREENING THEOREM HOLDS: "Up to one loop the only observables radiative corrections that grow with the Higgs mass are logarithmic".

Gauge interactions break the symmetry

$\Rightarrow$ we expect the $\rho=1$ relation to be broken at loop level

Since the custodial symmetry is also broken by Yukawa couplings we expect corrections to $\rho=1$ quadratic in the top mass (the heaviest quark)

Radiative corrections

The fundamental theorem that a gauge theory with SSB and Higgs mechanism is renormalizable was proven by 't Hooft

The renormalizability of the theory ensures that radiative corrections can reliably be computed

Status of the art : full one-loop corrections plus selected dominant two loop contributions

renormalization group improvements for large QED and QCD logs.

An important class of radiative corrections are those QUADRATIC IN M_{TOP}

Decoupling theorem

Diagrams with virtual heavy particles of mass M can be ignored at scales $q \ll M$ provided that the couplings do not grow with M and that the theory without the heavy particle is still consistent (renormalizable)

In the SM the limit $M_{TOP} \to \infty$ violates both conditions

$\Rightarrow$ $G_F M_{TOP}^2$ terms appear in various diagrams and radiative corrections are VERY SENSITIVE to the top mass

This is the reason why the top mass was predicted before discovery through EW precision tests.

EW tests

The way to proceed : choose a set of observables that are measured very precisely

$\Rightarrow$ M_Z , G_F , α_{em}

Start from tree-level relations to be modified by radiative corrections

$$\frac{4}{\sqrt{2}} G_F = \frac{g^2}{2M_W^2} \Rightarrow M_W^2 = \frac{e^2\sqrt{2}}{2\sin^2\vartheta \, 4G_F} \Rightarrow M_W^2 = \frac{\pi\alpha}{\sqrt{2}\, G_F \sin^2\vartheta}$$

define

$$M_W^2 \equiv \frac{\pi\alpha(M_Z)}{\sqrt{2}\, G_F \sin^2\vartheta_W (1-\underline{\Delta r_W})}$$

does not include pure QED effects

$$g_A = \frac{1}{2}$$

$$g_V \equiv -\frac{1}{2} + 2\sin^2\tilde{\vartheta}$$

$$\sin^2\tilde{\vartheta} = (1+\underline{\Delta k})\sin^2\vartheta_0$$

$$\sin^2\vartheta_0 \cos^2\vartheta_0 = \frac{\pi\alpha(M_Z)}{\sqrt{2}\, G_F M_Z^2}$$

ϑ_0 mixing angle WITHOUT rad corrections
that are not pure QED

$$\Gamma_e \equiv \frac{G_F M_Z^3}{6\pi\sqrt{2}} (1+\underline{\Delta \rho}) (g_V^2 + g_A^2)$$

$\Rightarrow$

$\Delta r_W \Leftrightarrow M_W$

$\Delta k \Leftrightarrow A_{FB}^{\mu}$

$\Delta \rho \Leftrightarrow \Gamma_e$

$$\begin{cases} \varepsilon_1 = \Delta\rho \\ \varepsilon_2 = \cos^2\vartheta\, \Delta\rho + \dfrac{\sin^2\vartheta}{\cos^2\vartheta - \sin^2\vartheta} \Delta r_W - 2\sin^2\vartheta\, \Delta k \\ \varepsilon_3 = \cos^2\vartheta\, \Delta\rho + (\cos^2\vartheta - \sin^2\vartheta)\Delta k \end{cases}$$

$\varepsilon_1 \sim m_{TOP}^2$

$\varepsilon_2, \varepsilon_3 \sim \log m_{TOP}$

$\varepsilon_2, \varepsilon_3 \Rightarrow$ used to assess new physics effects

3σ discrepancy between $\sin^2\tilde{\vartheta}$ measured by SLD (A_{LR}) and LEP (A_{FB}^{b})

Theoretical limits on m_H (29)

Let us assume that the SM is valid up to a scale Λ $\Rightarrow$ the Higgs sector is NOT asymptotically free and if m_H is too heavy we can hit the Landau pole at a scale $Q^2 < \Lambda$ $\Rightarrow$ this provides an upper limit on m_H as a function of Λ

Higgs self coupling

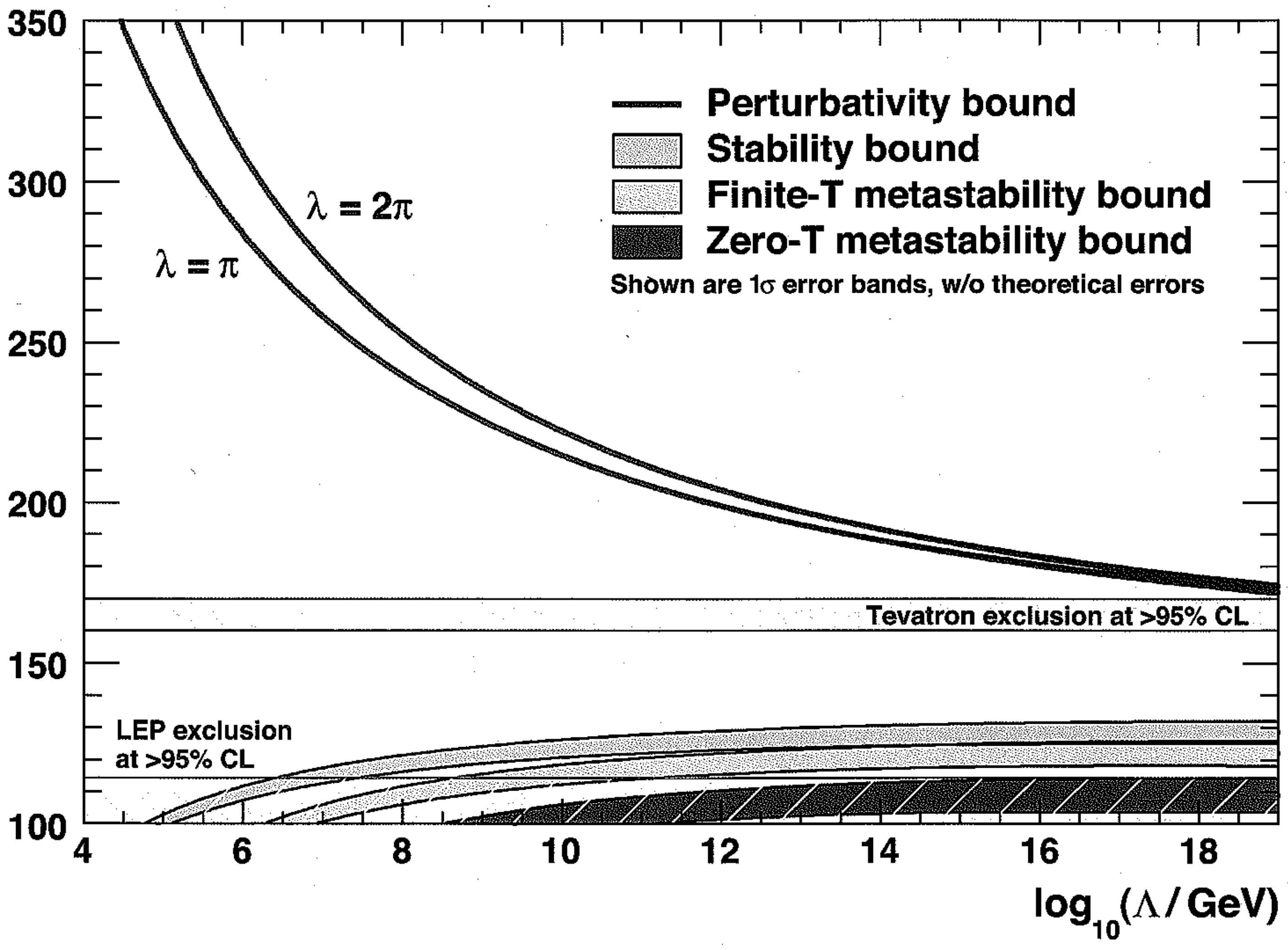
$$\frac{d\lambda}{dt} = \frac{3}{4\pi^2}\left(\lambda^2 + 3\lambda h_t^2 - 9 h_t^4 + \text{gauge terms}\right) \qquad t = \ln \Lambda / v$$

top Yukawa

If on the other hand the Higgs is light (and top heavy) then the quartic term in h_t dominates $\Rightarrow$ we get a vacuum instability

THE REGION OF ALLOWED HIGGS MASSES BECOMES SMALLER AND SMALLER AS Λ INCREASES

The present limits of a $m_H = 125$ GeV would suggest a Higgs at the border of the metastability region! (no problem however of the "vacuum lifetime" is longer than the lifetime of our universe)



SUMMARY AND OPEN QUESTIONS

The SM works remarkably well and no evidence of physics BSM has been discovered yet (except maybe in the neutrino sector!)

Naturalness problem

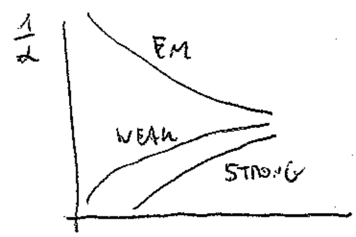
If the SM works up to a scale Λ radiative corrections to the Higgs mass are quadratic in m_H (no symmetry protects scalars!)

$$m_H^2 = m_{H^0}^2 + C\Lambda^2$$

$\Rightarrow$ If Λ is large the natural scale for m_H is the cut off itself unless we ensure a FINE TUNING of many order of magnitude

Unification of the couplings

By studying the evolution of the gauge couplings we see that in the SM they tend to reach the same value at $\Lambda_{GUT} \sim 10^{15}$ GeV



The unification is, nonetheless, not perfect

$\Rightarrow$ a more precise unification is obtained in the MSSM

Lep paradox

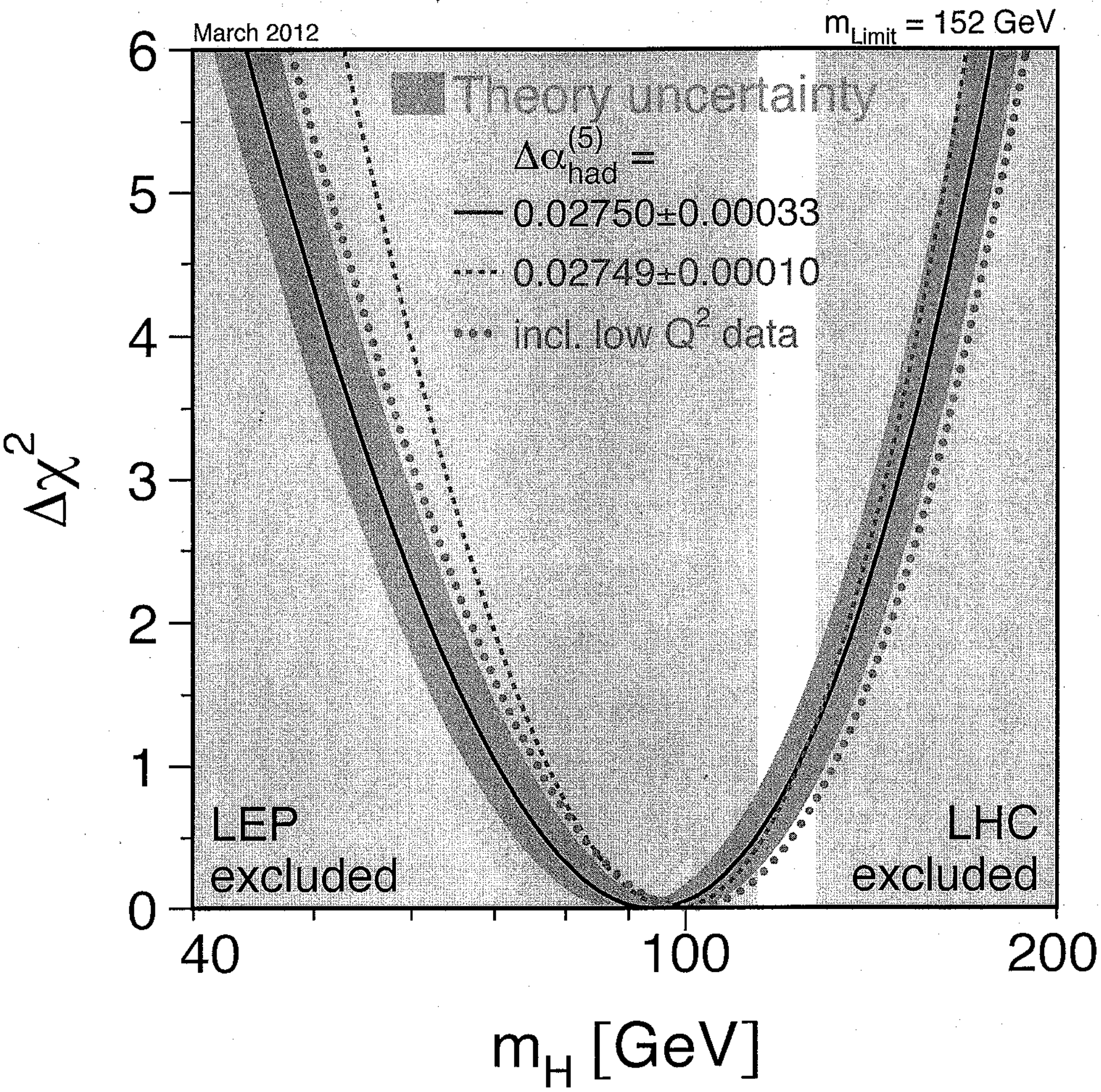
EW precision data suggest that the Higgs should be light $\Rightarrow$ either this happens with fine tuning, or new physics should really be closeby

$\Rightarrow$ but we did not see it yet! (and the LHC seems to confirm this statement)
at least for the moment

Maybe fine tuning is not a sensible guiding principle?

$\Rightarrow$ ANTHROPIC SOLUTION:

In short: we live in the only universe that made life possible



March 2012
m_{Limit} = 152 GeV
Theory uncertainty
$\Delta\alpha_{had}^{(5)}$ =
0.02750±0.00033
0.02749±0.00010
incl. low Q^2 data
LEP excluded
LHC excluded
$\Delta\chi^2$
m_H [GeV]
0
1
2
3
4
5
6
40
100
200