Particle Physics Phenomenology II: hadron collider physics

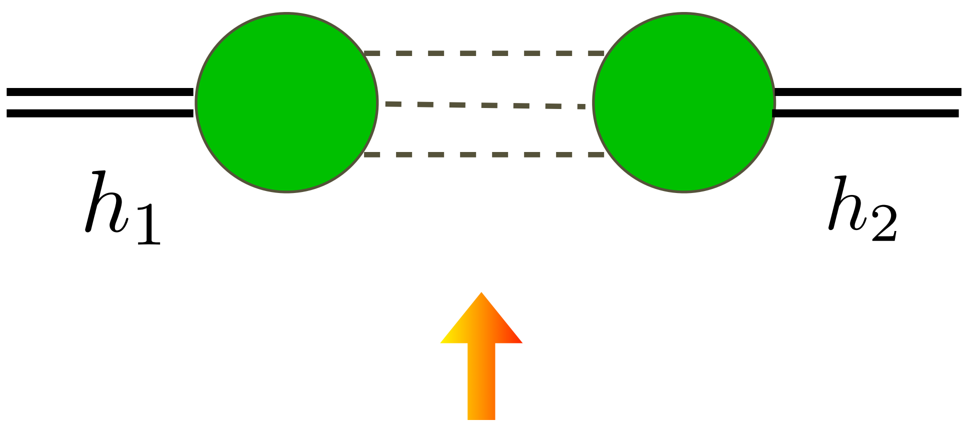
Massimiliano Grazzini

University of Zurich

may 8, 2012

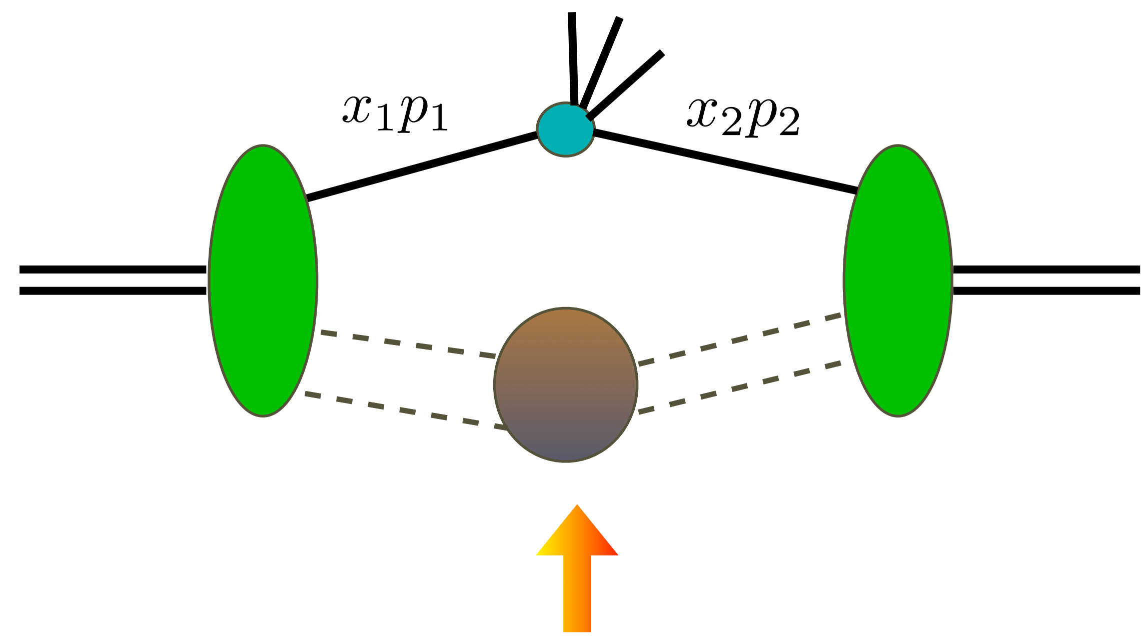
QCD at hadron colliders

In hadron collisions all phenomena are QCD related but we must distinguish between hard and soft processes



Production of low pt hadrons: most common events

Hard subprocess



Soft underlying event

Only hard scattering events can be controlled via the factorization theorem

Hard processes are identified by the presence of a hard scale Q

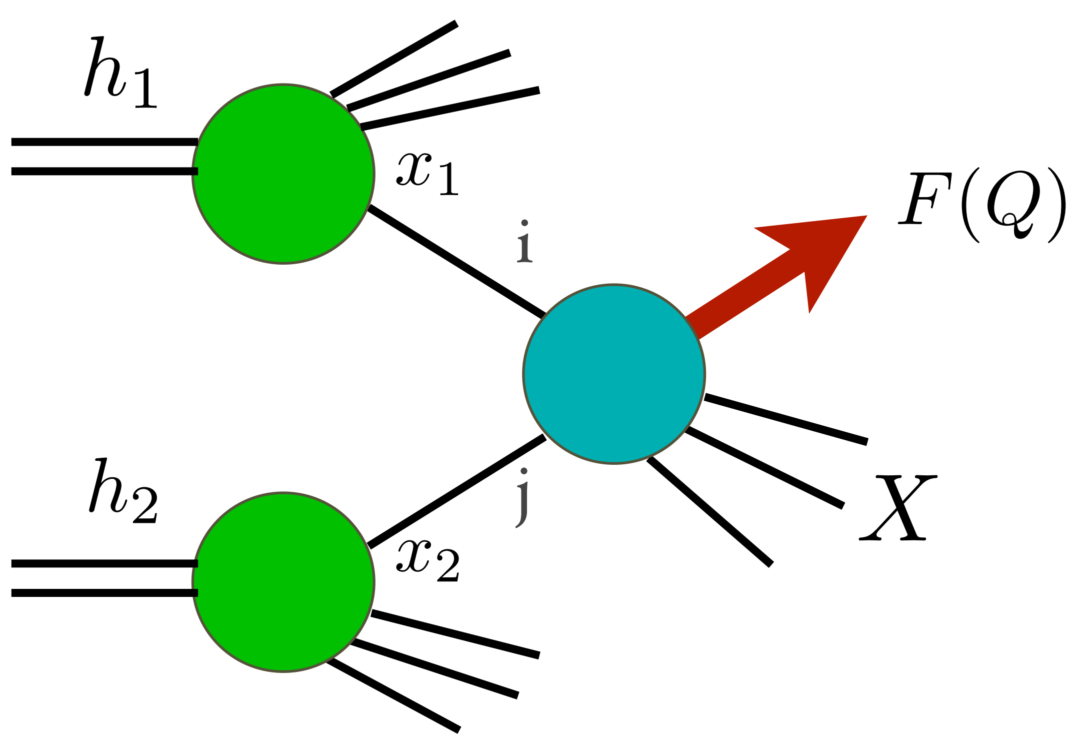
This can be for example the invariant mass of a lepton pair, the transverse momentum of a jet or of a heavy quark...

The corresponding cross section can be written as

$$\sigma(P_1, P_2) = \sum_{i,j} \int dx_1 dx_2 f_{i/h_1}(x_1, \mu_F^2) f_{j/h_2}(x_2, \mu_F^2) \hat{\sigma}_{ij}(p_1, p_2, \alpha_S(\mu_R), Q^2; \mu_F^2, \mu_R^2)$$

$p_1 = x_1 P_1$ $\qquad$ $p_2 = x_2 P_2$ $\qquad$ $f_{i/h}(x, \mu_F^2)$

Same parton densities measured in DIS !



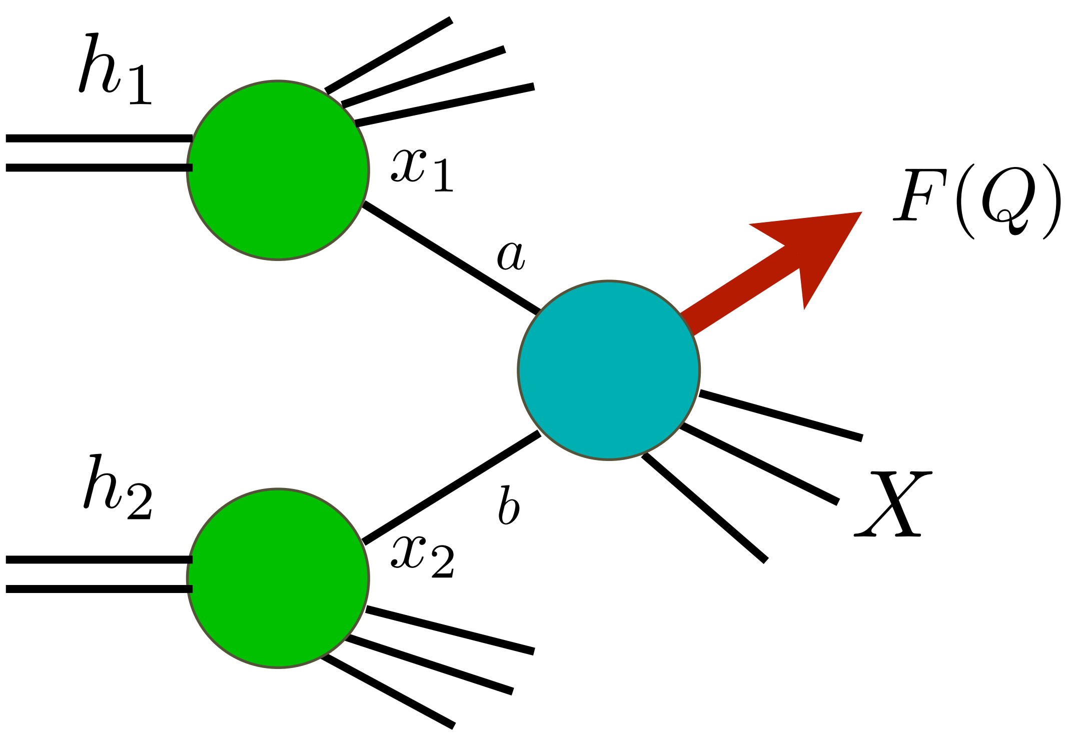
The partonic cross section can be computed in QCD perturbation theory as

$$\hat{\sigma}_{ij} = \alpha_S^k \sum_n \left(\frac{\alpha_S}{\pi}\right)^n \sigma_{ij}^n$$

Different hard processes will contribute with different leading powers k:

- Vector boson production: $k = 0$
- Jet production: $k = 2$

According to the factorization theorem, the initial state collinear singularities can be absorbed in the parton distribution functions as in the case of DIS



Note that the generally speaking the factorization theorem in hadron collisions does not have a solid proof as in DIS (where OPE can be advocated) !

The factorization scale μ_F is an arbitrary parameter
As in DIS, it can be thought as the scale that separates long and short distance physics

It should be chosen of the order of the hard scale Q
A similar argument works for the renormalization scale μ_R

If the calculation could be done to all orders, the physical cross section would not depend on μ_R and μ_F

In particular, the μ_F dependence of the parton distributions would be exactly compensated by that of the partonic cross section

Truncating the perturbative expansion at a given order n, the hadronic cross section has a residual scale dependence of order n+1

→ Variations of μ_R and μ_F around Q can give an idea of the size of uncalculated higher order contributions

Kinematics

The spectrum of the two hadrons provides two beams of incoming partons

The spectrum of longitudinal momenta is determined by the parton distributions

The centre of mass of the partonic interaction is normally boosted with respect to the laboratory frame

➡ It is useful to classify the final state according to variables that transform simply under longitudinal boosts

We introduce the rapidity y and the azimuthal angle ϕ

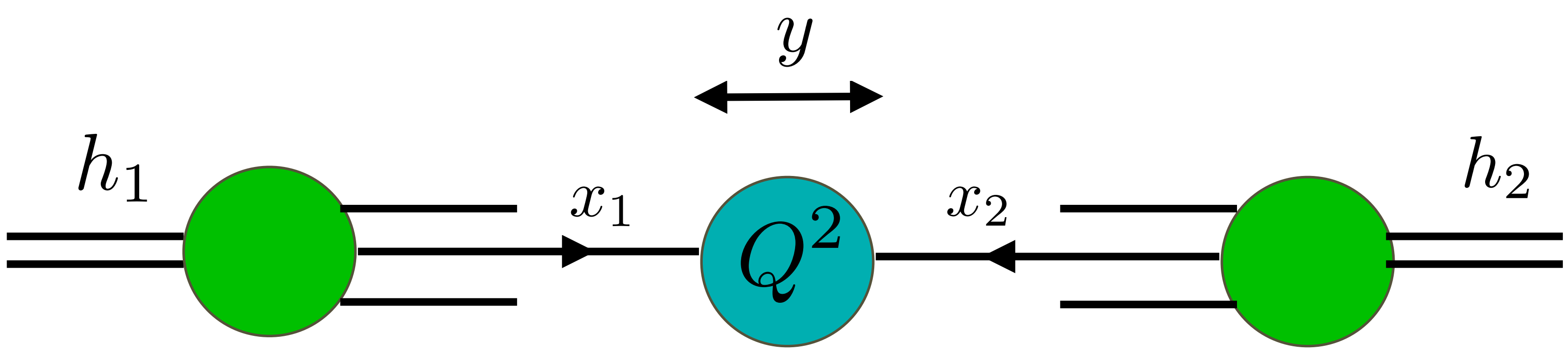
$$p^\mu = (E, p_x, p_y, p_z) = (m_T \cosh y, p_T \sin\phi, p_T \cos\phi, m_T \sinh y)$$

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z} \qquad m_T = \sqrt{m^2 + p_T^2}$$

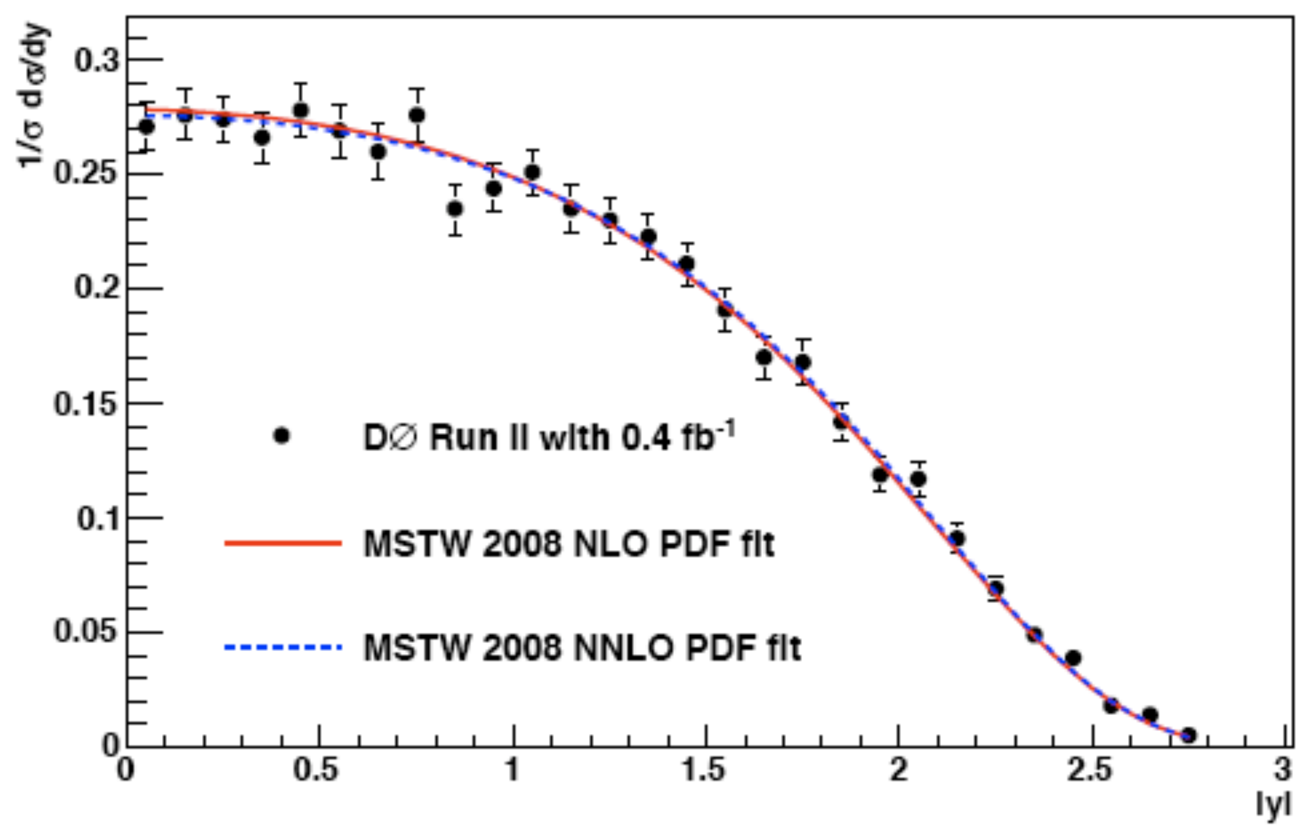
Rapidity differences are boost invariant

Varying Q and y → Sensitivity to different x_1, x_2

$$x_1 x_2 S = Q^2 \qquad x_{1,2} = Q/\sqrt{S}\, e^{\pm y}$$



At large rapidities we have two competitive effects:
- small x enhancement of gluon and sea quark distributions
- large x suppression



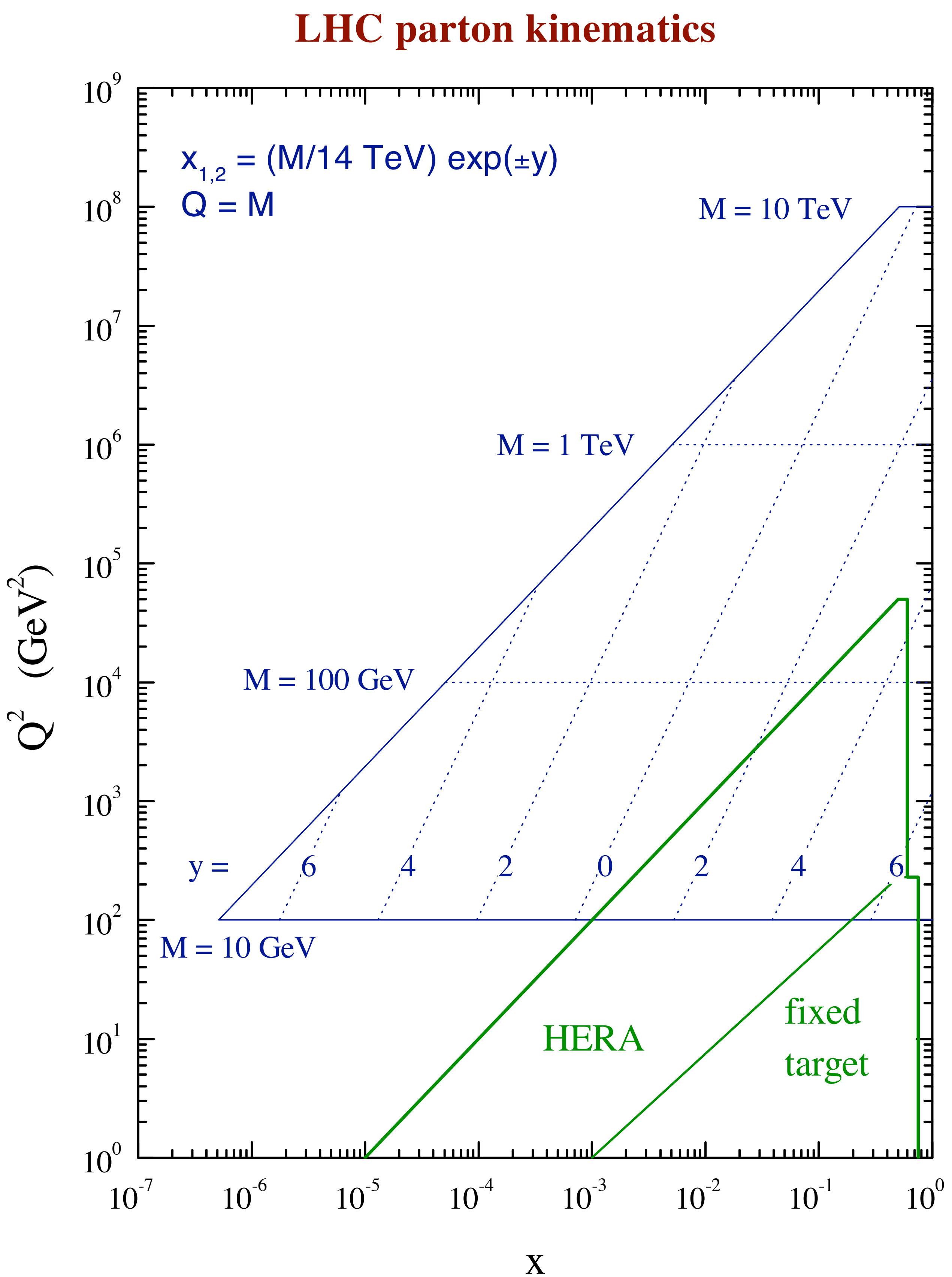
The large x suppression always "wins":

→ The bulk of the events is concentrated in the central rapidity region (y not too large)

In practice the rapidity is often replaced by the pseudorapidity

$$\eta = -\ln\tan(\theta/2)$$

It coincides with the rapidity
in the massless limit



Varying Q and y

→ Sensitivity to different x_1, x_2

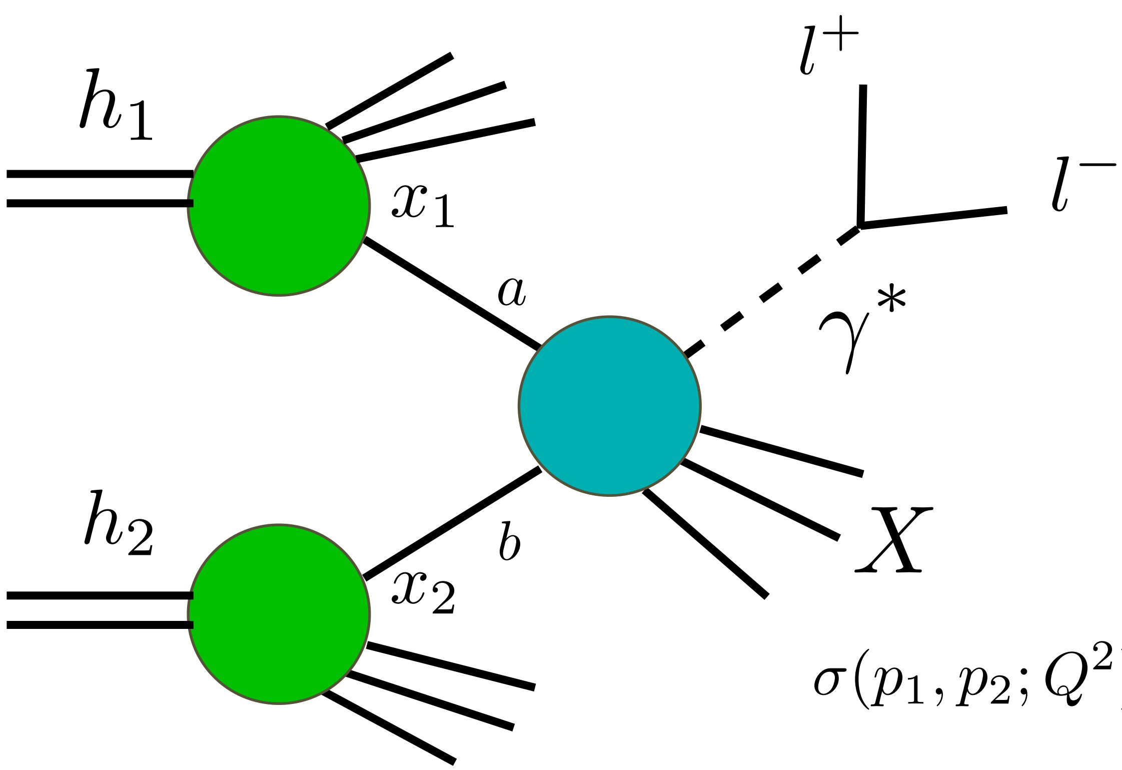
$$x_1 x_2 S = Q^2 \qquad x_{1,2} = Q/\sqrt{S}\, e^{\pm y}$$

LHC will probe a kinematical
region never reached before

The Drell-Yan process

The Drell-Yan mechanism was historically the first process where parton model ideas developed for DIS were applied to hadron collisions

Drell, Yan (1970)



Same parton densities measured in DIS !

$$\sigma(p_1, p_2; Q^2) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2\, f_{h_1,a}(x_1, \mu_F^2)\, f_{h_2,b}(x_2, \mu_F^2)$$

$$\times \hat{\sigma}_{ab}(x_1 p_1, x_2 p_2, \alpha_S(Q^2), \mu_F^2)$$

The hard scale is given by the invariant mass Q^2 of the lepton pair

It lead to the discovery of W and Z bosons at CERN !

Volume 122B, number 1 PHYSICS LETTERS 24 February 1983

EXPERIMENTAL OBSERVATION OF ISOLATED LARGE TRANSVERSE ENERGY ELECTRONS WITH ASSOCIATED MISSING ENERGY AT $\sqrt{s}$ = 540 GeV

UA1 Collaboration, CERN, Geneva, Switzerland

G. ARNISON[j], A. ASTBURY[j], B. AUBERT[b], C. BACCI[i], G. BAUER[1], A. BÉZAGUET[d], R. BÖCK[d], T.J.V. BOWCOCK[f], M. CALVETTI[d], T. CARROLL[d], P. CATZ[b], P. CENNINI[d], S. CENTRO[d], F. CERADINI[d], S. CITTOLIN[d], D. CLINE[1], C. COCHET[k], J. COLAS[b], M. CORDEN[c], D. DALLMAN[d], M. DeBEER[k], M. DELLA NEGRA[b], M. DEMOULIN[d], D. DENEGRI[k], A. Di CIACCIO[i], D. DiBITONTO[d], L. DOBRZYNSKI[g], J.D. DOWELL[c], M. EDWARDS[c], K. EGGERT[a], E. EISENHANDLER[f], N. ELLIS[d], P. ERHARD[a], H. FAISSNER[a], G. FONTAINE[g], R. FREY[h], R. FRÜHWIRTH[l], J. GARVEY[c], S. GEER[g], C. GHESQUIÈRE[g], P. GHEZ[b], K.L. GIBONI[a], W.R. GIBSON[f], Y. GIRAUD-HÉRAUD[g], A. GIVERNAUD[k], A. GONIDEC[b], G. GRAYER[j], P. GUTIERREZ[h], T. HANSL-KOZANECKA[a], W.J. HAYNES[j], L.O. HERTZBERGER[2], C. HODGES[h], D. HOFFMANN[a], H. HOFFMANN[d], D.J. HOLTHUIZEN[2], R.J. HOMER[c], A. HONMA[f], W. JANK[d], G. JORAT[d], P.I.P. KALMUS[f], V. KARIMÄKI[e], R. KEELER[f], I. KENYON[c], A. KERNAN[h], R. KINNUNEN[e], H. KOWALSKI[d], W. KOZANECKI[h], D. KRYN[d], F. LACAVA[d], J.-P. LAUGIER[k], J.-P. LEES[b], H. LEHMANN[a], K. LEUCHS[a], A. LÉVÊQUE[k], D. LINGLIN[b], E. LOCCI[k], M. LORET[k], J.-J. MALOSSE[k], T. MARKIEWICZ[d], G. MAURIN[d], T. McMAHON[c], J.-P. MENDIBURU[g], M.-N. MINARD[b], M. MORICCA[i], H. MUIRHEAD[d], F. MULLER[d], A.K. NANDI[j], L. NAUMANN[d], A. NORTON[d], A. ORKIN-LECOURTOIS[g], L. PAOLUZI[i], G. PETRUCCI[d], G. PIANO MORTARI[i], M. PIMIÄ[e], A. PLACCI[d], E. RADERMACHER[a], J. RANSDELL[h], H. REITHLER[a], J.-P. REVOL[d], J. RICH[k], M. RIJSSENBEEK[d], C. ROBERTS[j], J. ROHLF[d], P. ROSSI[d], C. RUBBIA[d], B. SADOULET[d], G. SAJOT[g], G. SALVI[f], G. SALVINI[i], J. SASS[k], J. SAUDRAIX[k], A. SAVOY-NAVARRO[k], D. SCHINZEL[f], W. SCOTT[j], T.P. SHAH[j], M. SPIRO[k], J. STRAUSS[l], K. SUMOROK[c], F. SZONCSO[l], D. SMITH[h], C. TAO[d], G. THOMPSON[f], J. TIMMER[d], E. TSCHESLOG[a], J. TUOMINIEMI[e], S. Van der MEER[d], J.-P. VIALLE[d], J. VRANA[g], V. VUILLEMIN[d], H.D. WAHL[l], P. WATKINS[c], J. WILSON[c], Y.G. XIE[d], M. YVERT[b] and E. ZURFLUH[d]

Aachen [a]–Annecy (LAPP) [b]–Birmingham [c]–CERN [d]–Helsinki [e]–Queen Mary College, London [f]–Paris (Coll. de France) [g]–Riverside [h]–Rome [i]–Rutherford Appleton Lab. [j]–Saclay (CEN) [k]–Vienna [l] Collaboration

Received 23 January 1983

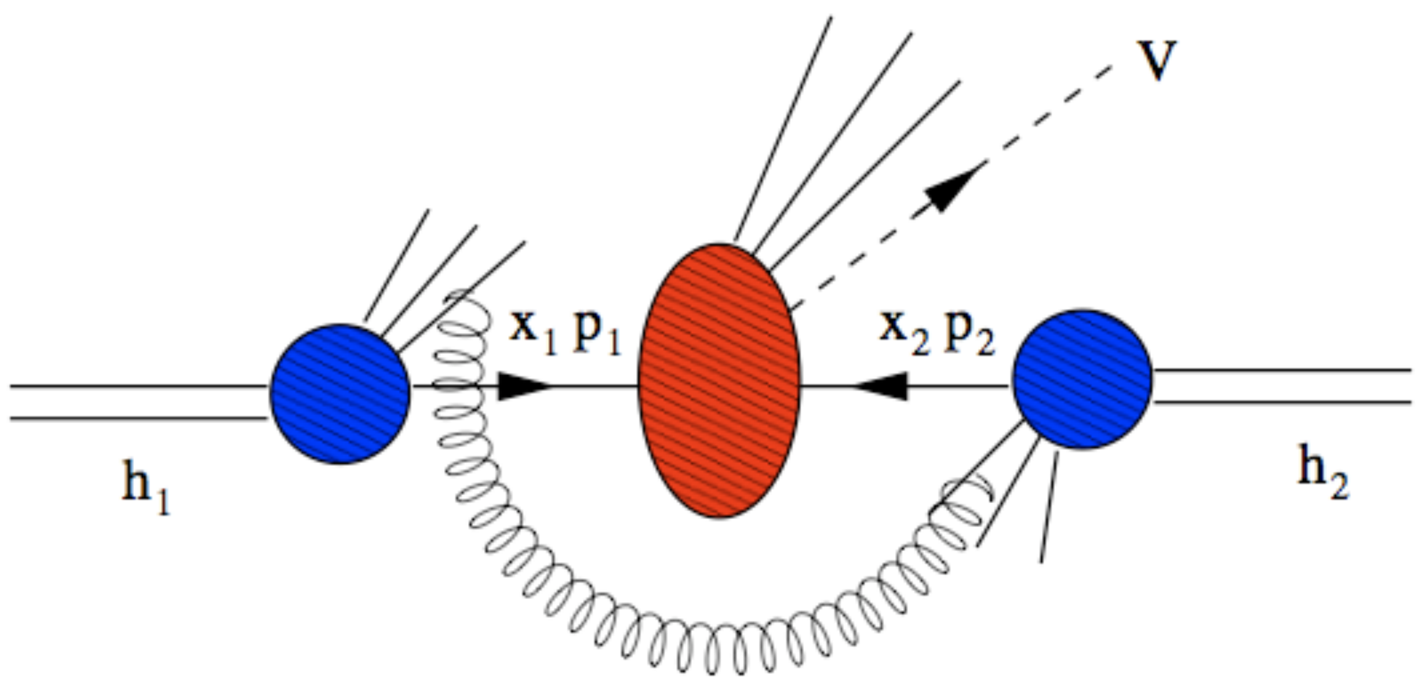
We report the results of two searches made on data recorded at the CERN SPS Proton–Antiproton Collider: one for isolated large-E_T electrons, the other for large-E_T neutrinos using the technique of missing transverse energy. Both searches converge to the same events, which have the signature of a two-body decay of a particle of mass ~80 GeV/c^2. The topology as well as the number of events fits well the hypothesis that they are produced by the process $\bar{p} + p \to W^{\pm} + X$, with $W^{\pm} \to e^{\pm} + \nu$; where $W^{\pm}$ is the Intermediate Vector Boson postulated by the unified theory of weak and electromagnetic interactions.

[1] University of Wisconsin, Madison, WI, USA.
[2] NIKHEF, Amsterdam, The Netherlands.

Proof of factorization:

similar to DIS: absorb initial state collinear divergences into a redefinition of parton distributions but....

The proof that soft gluons do not spoil factorization is very difficult: soft gluons (large wavelenght) can transfer colour information between the two initial state hadrons



Explicit calculations have shown that factorization breaking effects are present but are power suppressed in the high energy limit

Intuitive interpretation:

classical potential of an electric charge e moving with velocity v along the z axis

$$A^\mu = \frac{e}{[(z-vt)^2 + (1-v^2)(x^2+y^2)]^{1/2}}(1,0,0,v)$$

But when $v \to 1$ we have $A^\mu \sim \dfrac{e}{|z-vt|}(1,0,0,1)$

➡ the charged particle generates a potential extending along all the z axis

But when $z \neq vt$ $A^\mu \sim \dfrac{\partial}{\partial x_\mu} e \ln|z-vt|$ pure gauge !

The field $F_{\mu\nu}$ is not long-range but is localized on $z \sim vt$

$$\hat{\sigma}(q(p_1)\bar{q}(p_2) \to l^+ l^-) = \frac{4}{3}\pi \frac{\alpha^2}{\hat{s}} \frac{1}{N} Q_q^2$$

QED limit: $\sigma = \frac{4}{3}\pi\frac{\alpha^2}{\hat{s}}$

Average over number of colours

Quark electric charge

$$\frac{d\hat{\sigma}}{dM^2} = \frac{\sigma_0}{N} Q_q^2\, \delta(\hat{s} - M^2) \qquad \sigma_0 = \frac{4}{3}\pi\frac{\alpha^2}{M^2}$$

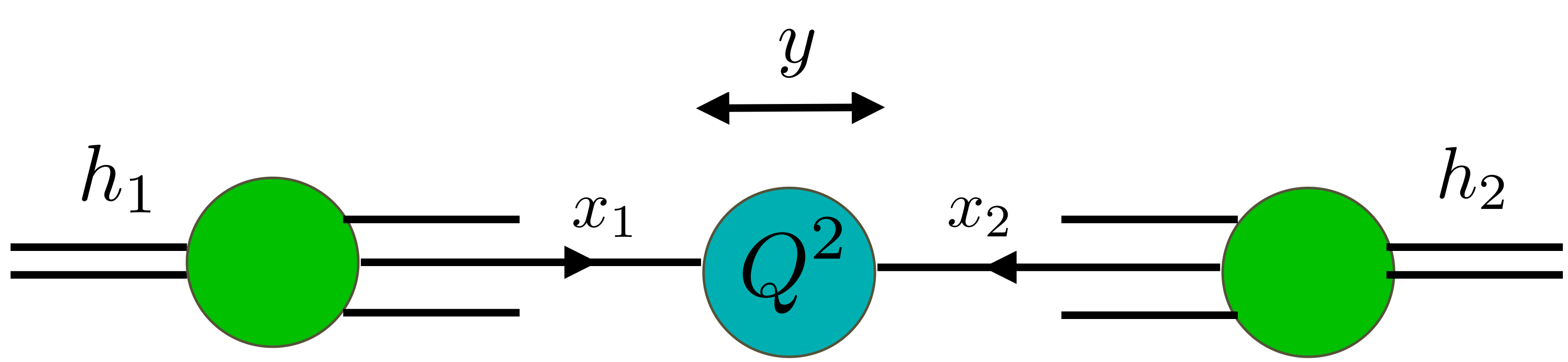
$$p_1 = \frac{\sqrt{s}}{2}(x_1, 0, 0, x_1)$$

$$p_2 = \frac{\sqrt{s}}{2}(x_2, 0, 0, -x_2)$$

$$y = \frac{1}{2}\ln\frac{E+p_z}{E-p_z} = \frac{1}{2}\ln\frac{x_1}{x_2}$$

$$x_1 = \sqrt{\tau}\exp(y)$$

$$x_2 = \sqrt{\tau}\exp(-y)$$



Scaling

In the parton model the parton distributions functions are independent of the scale

→ by constructing an adimensional quantity the Drell-Yan cross section exhibits scaling in the variable $\tau = M^2/s$

$$M^4 \frac{d\sigma}{dM^2} = \frac{4}{3}\pi\frac{\alpha^2}{N}\tau \int_0^1 dx_1 dx_2 \delta(x_1 x_2 - \tau) \sum_q Q_q^2 (f_q(x_1) f_{\bar{q}}(x_2) + (q \leftrightarrow \bar{q})) = \frac{4}{3}\pi\frac{\alpha^2}{N}\tau \mathcal{F}(\tau)$$

This scaling is completely analogous to the Bjorken scaling of DIS structure functions and is verified experimentally to a good approximation

Note that to test it one has to study $M^4 \frac{d\sigma}{dM^2}$ at fixed τ

$$\frac{d^2\sigma}{dM^2dy} = \frac{\sigma_0}{N\,s}\left[\sum_q Q_q^2\left(f_q(x_1)f_{\bar{q}}(x_2) + (q \leftrightarrow \bar{q})\right)\right]$$

$$x_1 = \sqrt{\tau}\exp(y)$$
$$x_2 = \sqrt{\tau}\exp(-y)$$

The parton model neglects parton transverse momenta

➡ Lepton pair has zero transverse momentum in LO QCD

Assume:

$$dx\,f(x) \rightarrow dk_T^2\,dx\,P(\mathbf{k_T}, x)$$ with $$\int d^2k_T P(\mathbf{k_T}, x) = f(x)$$

Consider a simple model in which: $P(\mathbf{k_T}, x) = h(\mathbf{k_T})f(x)$

$$\frac{1}{\sigma}\frac{d^2\sigma}{d^2p_T} = \int d^2k_{T1}d^2k_{T2}\,\delta^{(2)}(\mathbf{k_{T1}} + \mathbf{k_{T2}} - \mathbf{p_T})h(\mathbf{k_{T1}})h(\mathbf{k_{T2}})$$

Assuming a Gaussian distribution

$$h(\mathbf{k_T}) = \frac{b}{\pi}\exp(-b\,k_T^2)$$

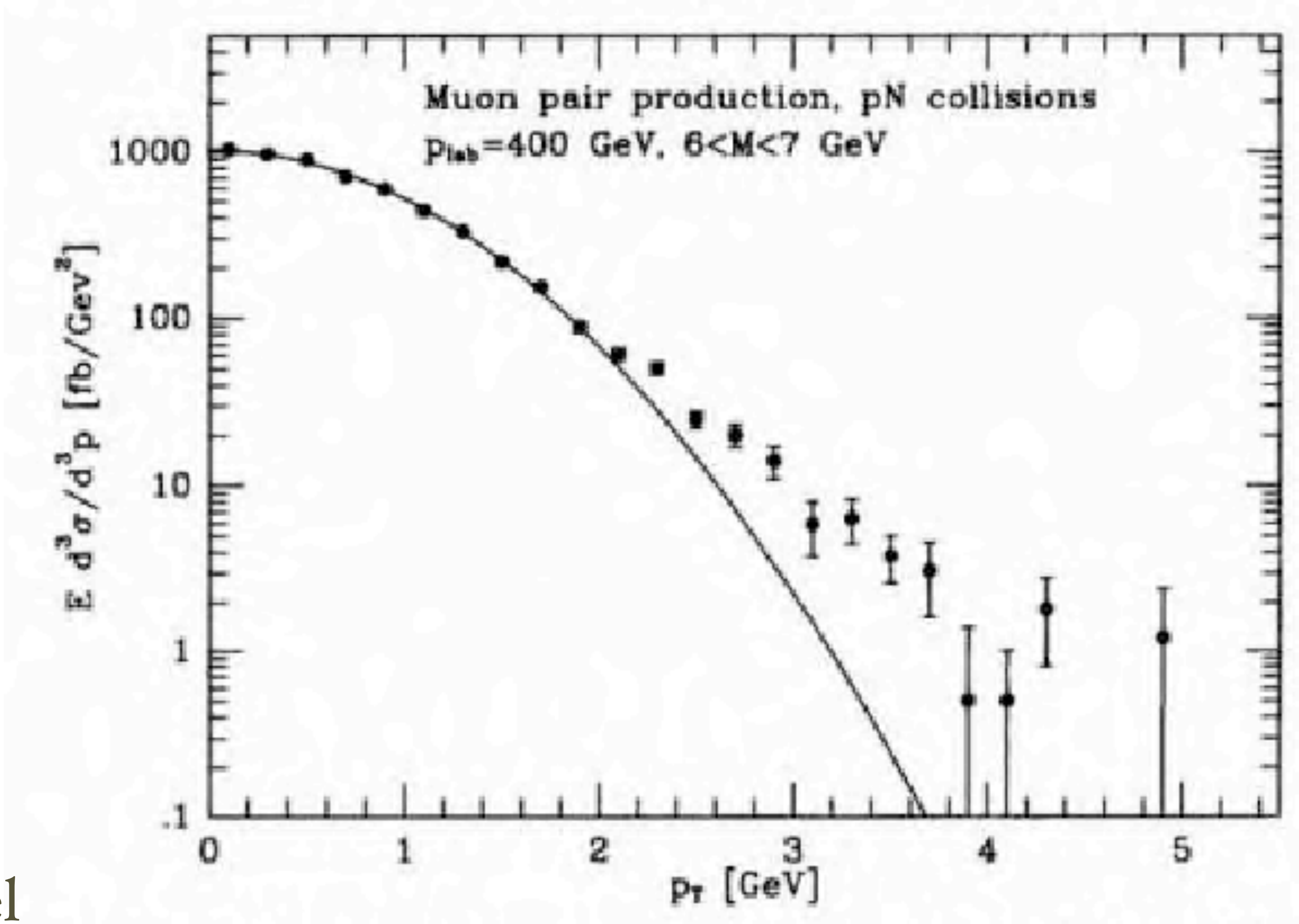
Dilepton spectrum from the CFS collaboration (1981)

the data correspond to

$$\langle k_T \rangle = \sqrt{\pi/4b} \sim 760\,\mathrm{MeV}$$

indeed of the order of the typical hadronic mass scale !

Historically the relative abundance of Drell-Yan lepton pairs with large transverse momenta provided one of the evidences that the parton model was incomplete

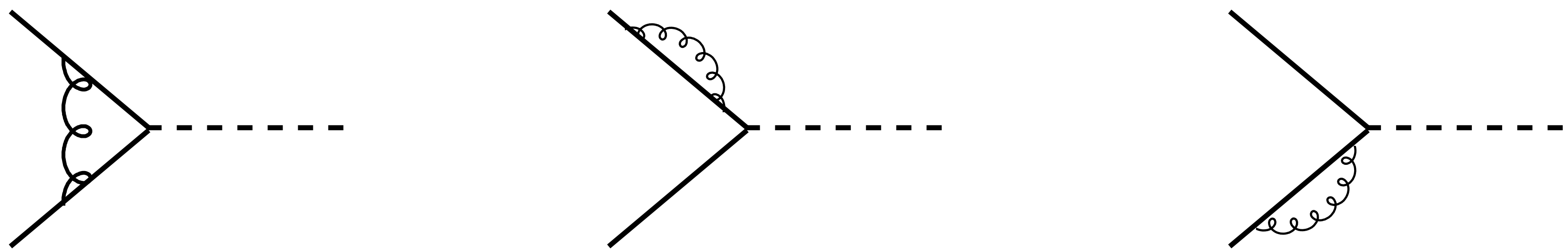


Transverse momentum is not generated only by “intrinsic” motion of the quarks in the hadrons but also by hard gluon radiation

$$\frac{d^2\sigma}{d^2p_T} \sim \frac{\alpha_S(p_T)}{p_T^n}$$

QCD corrections

virtual



real

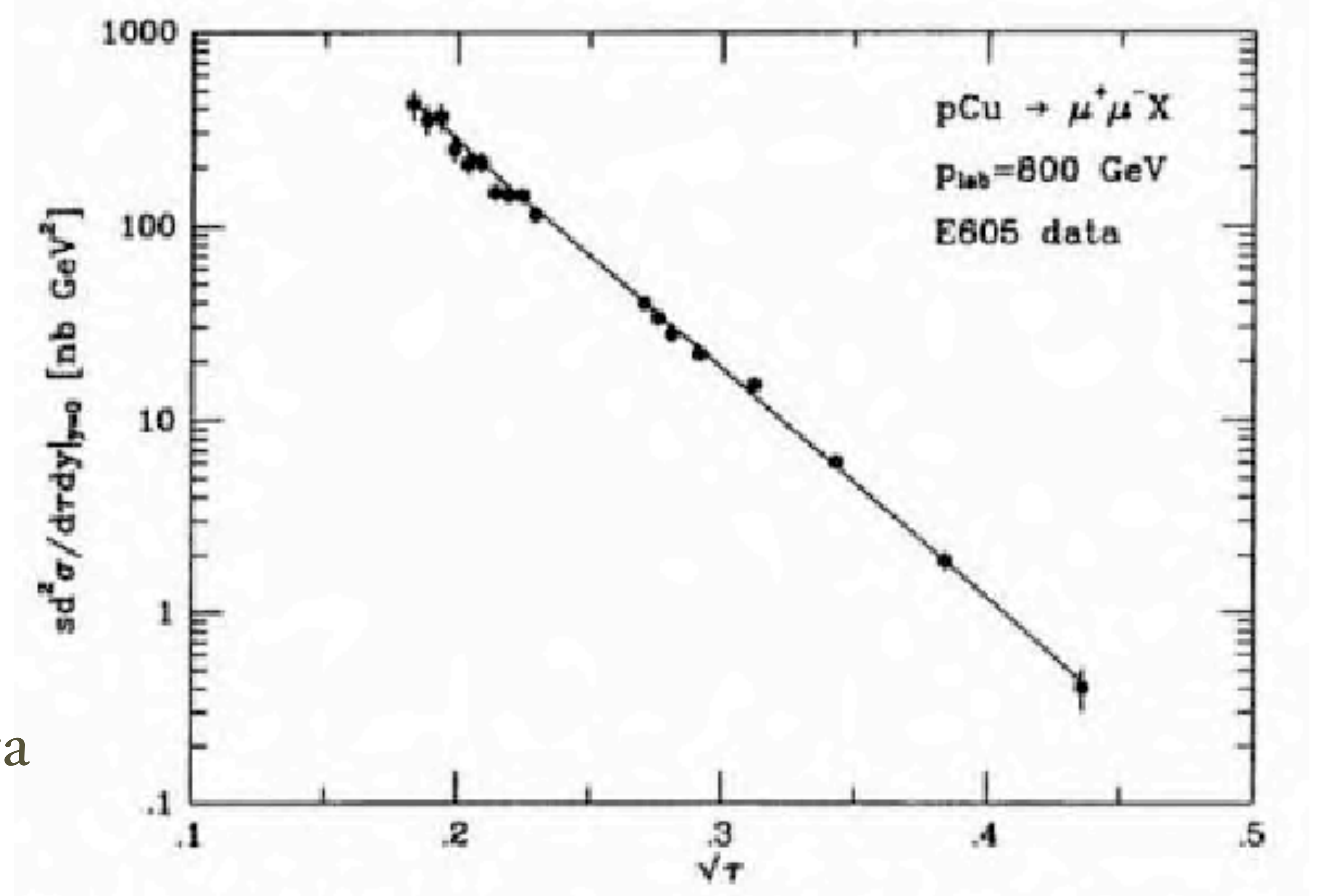
NLO corrections are at the 30-40% level

NNLO corrections are also known: they are at the few percent level

R.Hamberg, W.Van Neerven, T.Matsuura (1991)

Comparison with the data

The Drell-Yan cross section has been measured by a variety of experiments with different beams, targets and energies



Several informations can be obtained from the Drell-Yan data

Low mass lepton pair production at high energies is sensitive to the small x behavior

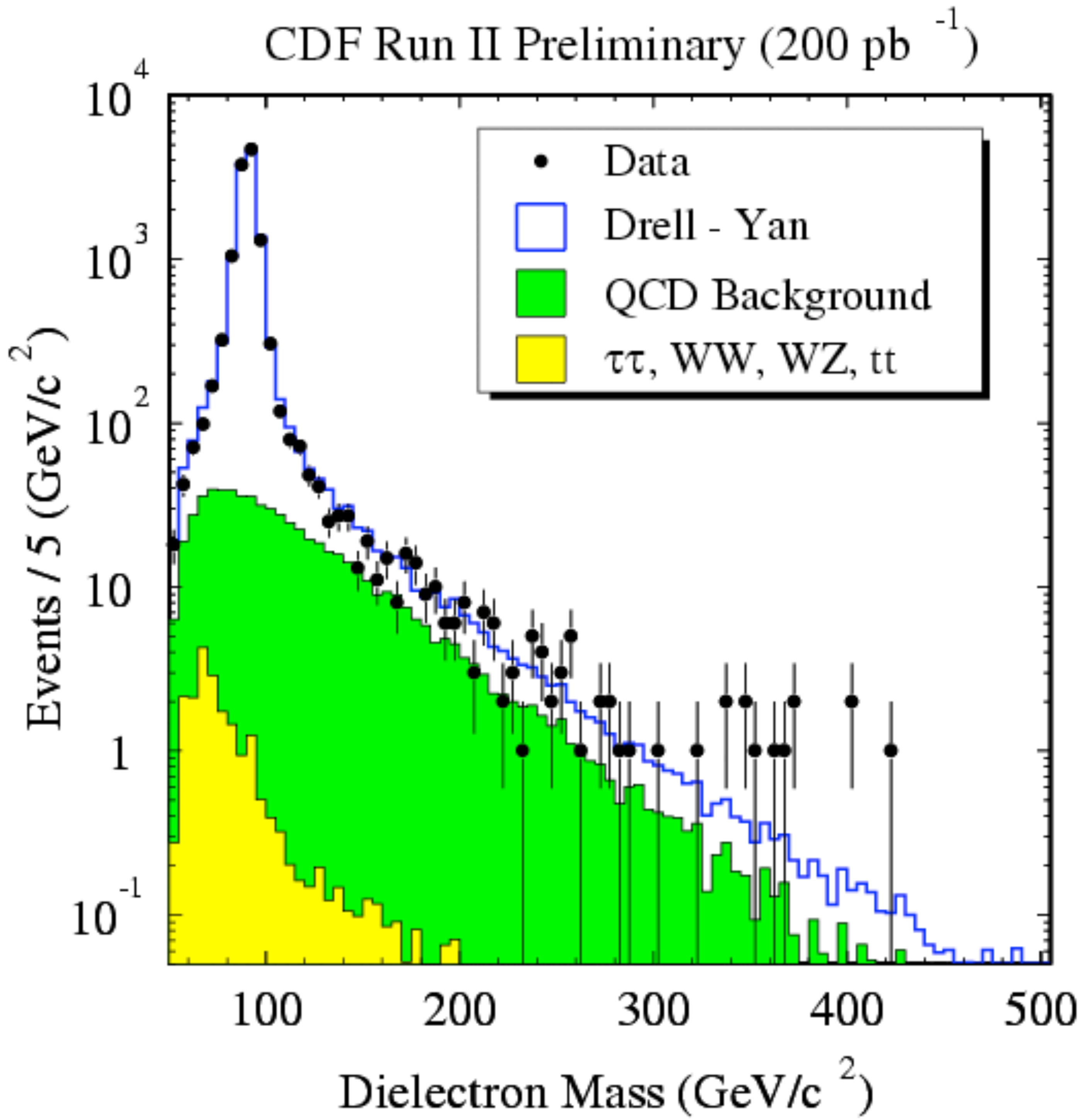
In pp collisions the cross section is sensitive to the sea quark distributions → complementary information to DIS

At higher energies the photon contribution must be supplemented with Z exchange

In practice lepton pair production around $M \sim m_Z$ is often analyzed using the narrow width approximation

$$\frac{1}{(\hat{s}-m_Z)^2+m_Z^2\Gamma_Z^2} \sim \frac{\pi}{m_Z\Gamma_Z}\delta(\hat{s}-m_Z^2)$$

The normalization is fixed by the condition that the two distributions have the same integral



W production: jacobian peak

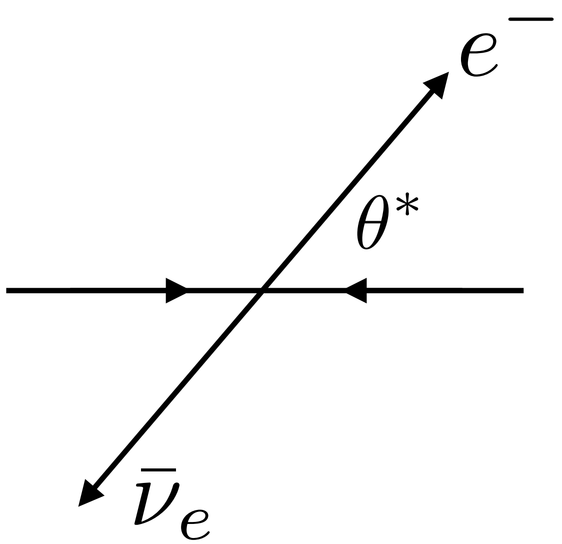
Since in the $W \to l\nu$ decay the neutrino momentum is not reconstructed the W invariant mass cannot be measured

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta^*} = \frac{3}{8}(1+\cos^2\theta^*)$$

angular distribution of the charged lepton in the W rest frame

The transverse momentum of the charged lepton carries information on m_W

At LO, however the W has zero transverse momentum



$$\cos\theta^* = \left(1 - \frac{4p_{Te}^2}{m_W^2}\right)^{1/2}$$

$$\frac{1}{\sigma}\frac{d\sigma}{dp_{Te}^2} = \frac{3}{m_W^2}\left(1 - \frac{4p_{Te}^2}{m_W^2}\right)^{-1/2}\left(1 - \frac{2p_{Te}^2}{m_W^2}\right)$$

strong peak at $p_{Te} = m_W/2$ (Jacobian peak)

In practice the peak is smeared by finite-width effects and QCD radiation

W production: transverse mass

Define now $m_T = \sqrt{2p^l_T p^{\rm miss}_T(1-\cos\phi)}$

azimuthal angle between electron and neutrino momenta

At LO $\phi = \pi$ and $p_{Te} = p^{\rm miss}_T$

imply $m_T = 2p_{Te}$

The transverse mass distribution has also a jacobian peak at $m_T = m_W$

W transverse mass distribution measured by the CDF collaboration (1995)

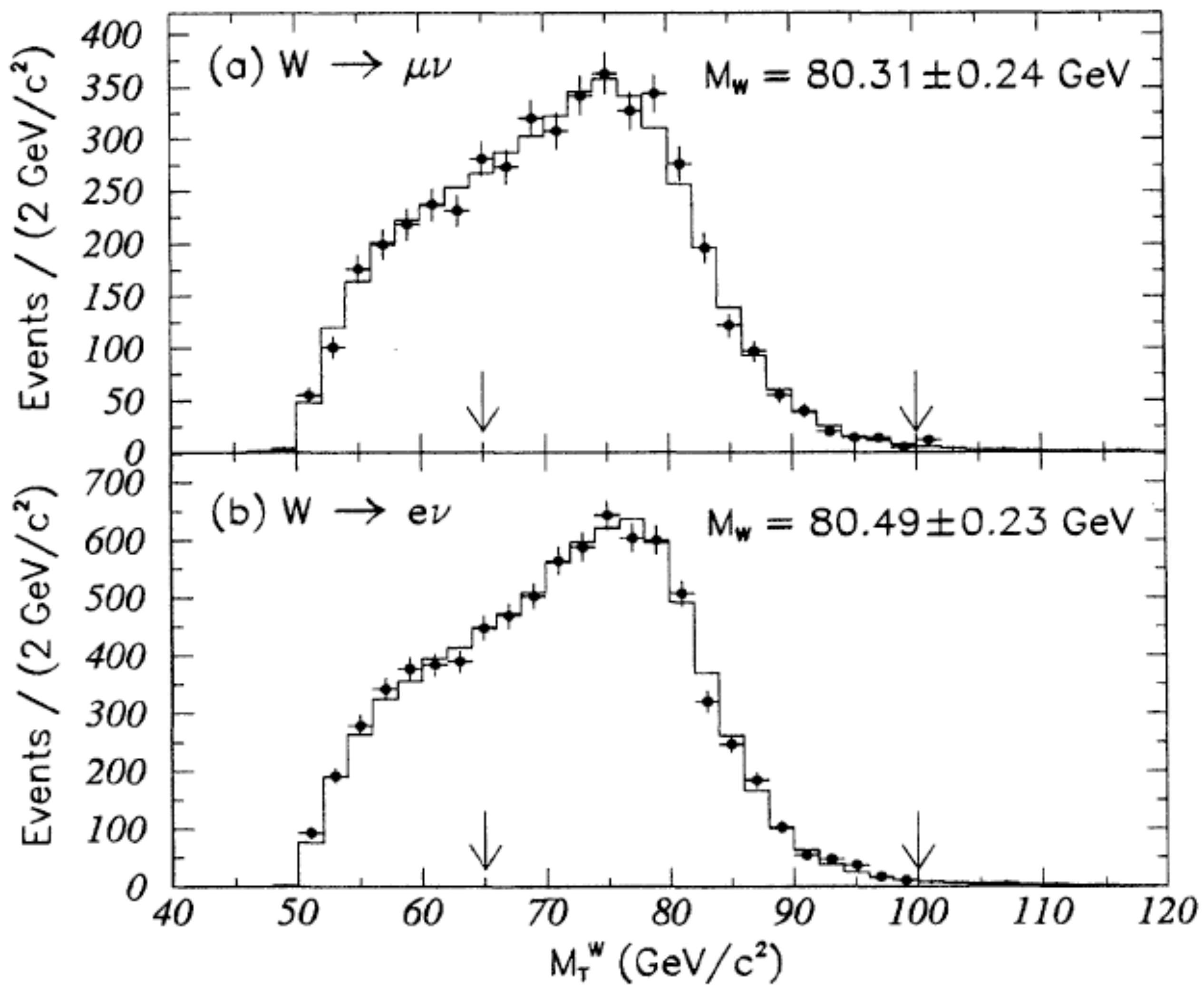


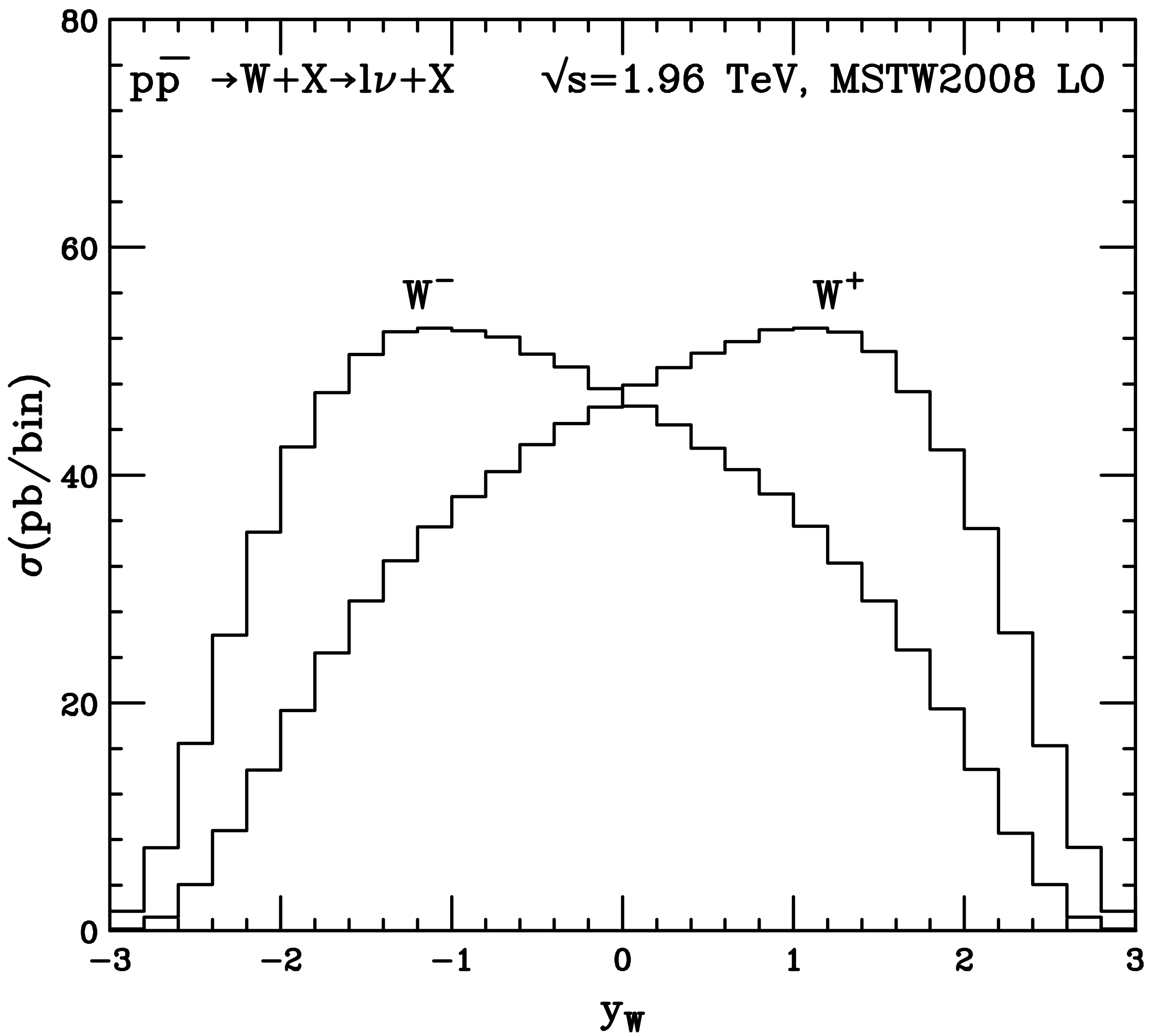
FIG. 2. Transverse mass spectra for (a) $W\to\mu\nu$ and (b) $W\to e\nu$ decays. The arrows delimit the fit region.

The advantage of the transverse mass is that it is less sensitive to the W transverse momentum with respect to p_{Te}

NB: If p^W_T is small $p_{Te,\nu} = \pm p + p^W_T/2$ leave the transverse mass invariant to first order

W charge asymmetry

An important observable in W hadroproduction is the asymmetry in the rapidity distributions of the W bosons



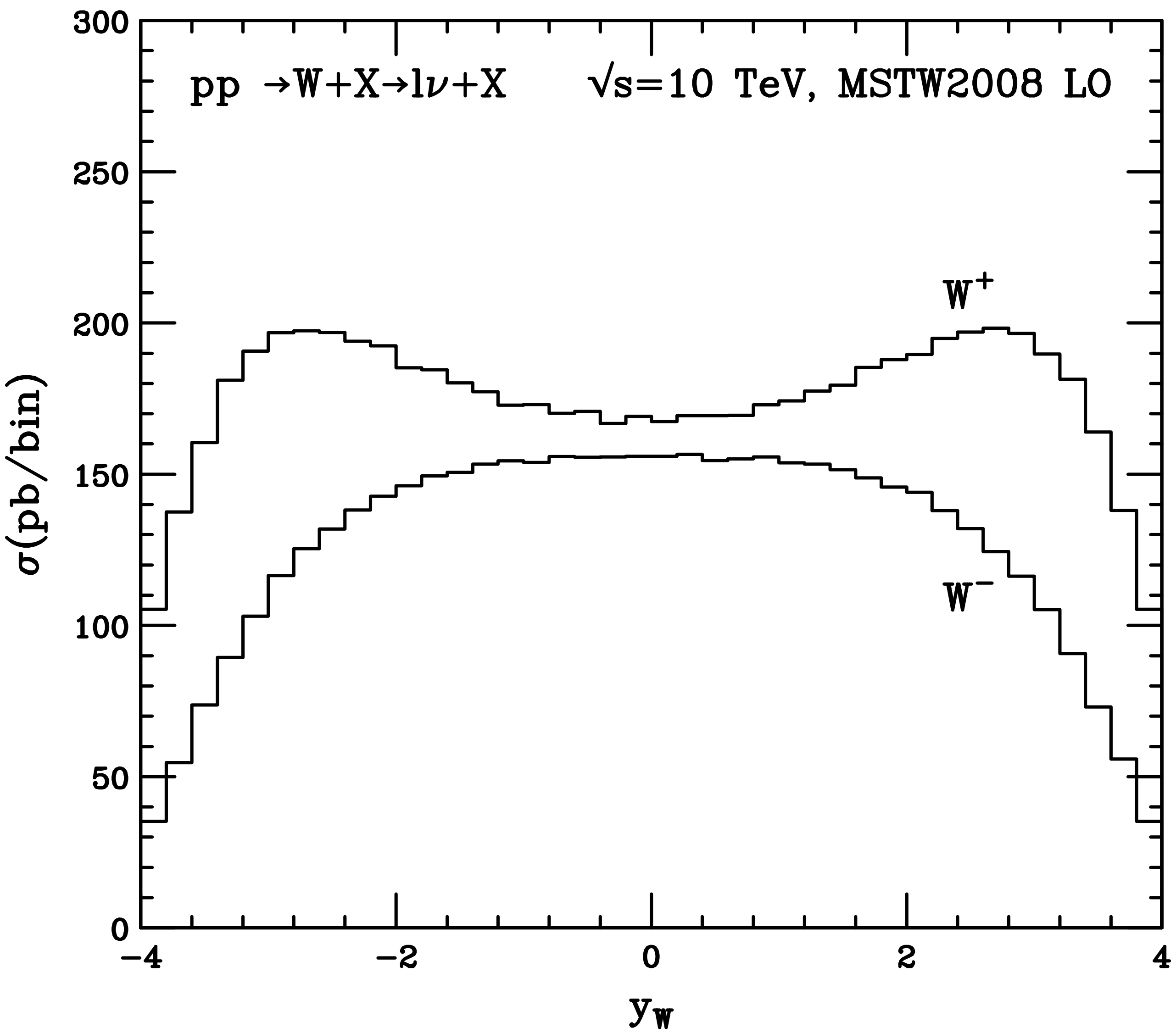
$$A(y_W) = \frac{\frac{d\sigma(W^+)}{dy_W} - \frac{d\sigma(W^-)}{dy_W}}{\frac{d\sigma(W^+)}{dy_W} + \frac{d\sigma(W^-)}{dy_W}}$$

In p$\bar{p}$ collisions the W^+ and W^- are produced with equal rates but W^+ (W^-) is produced mainly in the proton (antiproton) direction

These asymmetries are mainly due to the fact that, on average, the u quark carries more proton momentum fraction than the d quark

W charge asymmetry

An important observable in W hadroproduction is the asymmetry in the rapidity distributions of the W bosons



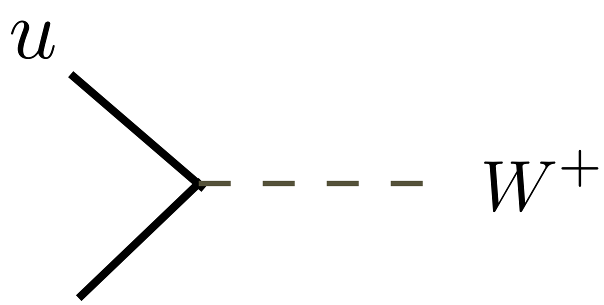
$$A(y_W) = \frac{\frac{d\sigma(W^+)}{dy_W} - \frac{d\sigma(W^-)}{dy_W}}{\frac{d\sigma(W^+)}{dy_W} + \frac{d\sigma(W^-)}{dy_W}}$$

In pp collisions the W^+ and W^- are produced with different rates but W^+ and W^- rapidity distributions are forward-backward symmetric W^- distribution is central, whereas W^+ is produced at larger rapidities

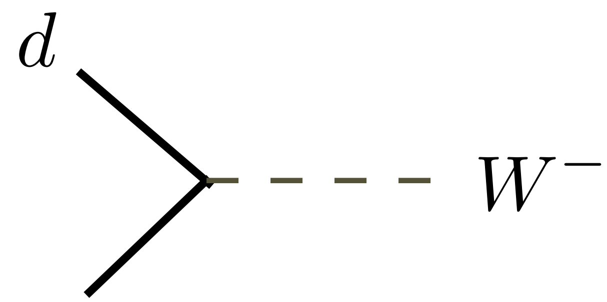
These asymmetries are mainly due to the fact that, on average, the u quark carries more proton momentum fraction than the d quark

W charge asymmetry

In $p\bar{p}$ collisions:



$\bar{d} = d$ in the proton



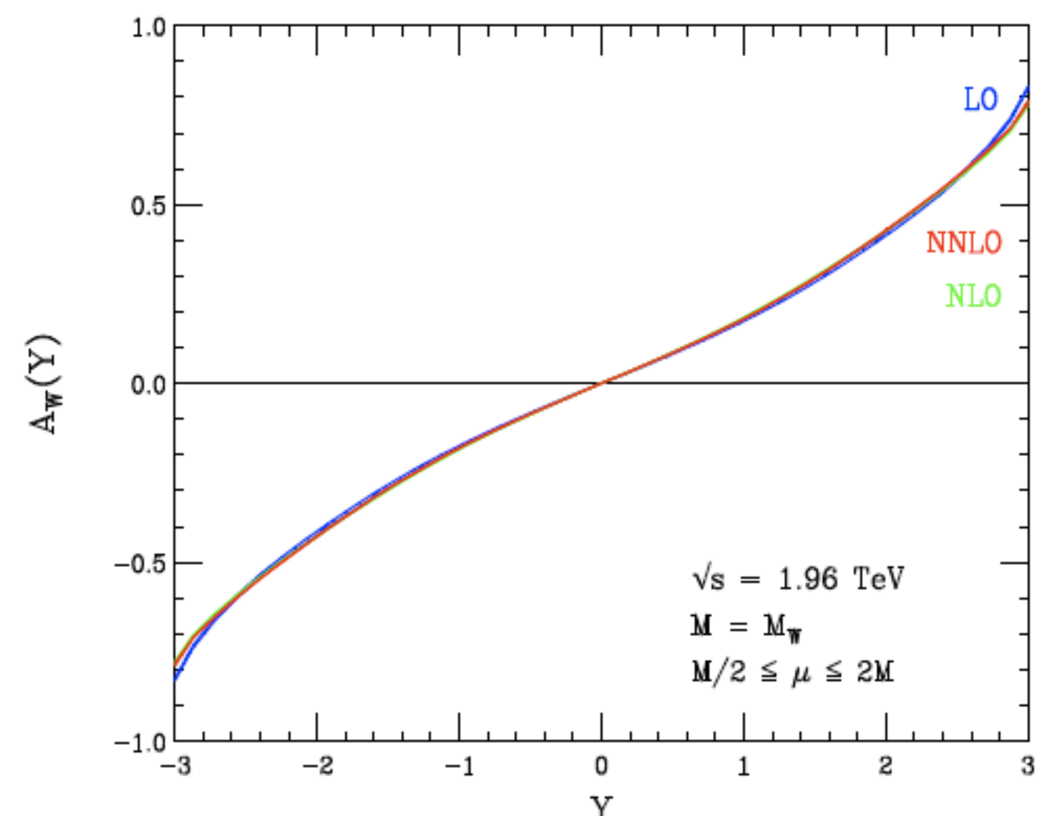
$\bar{u} = u$ in the proton

If u in the proton is faster than d $(u(x) > d(x))$

$W^+(W^-)$ produced mainly in p (pbar) direction

The W asymmetry $A(y) = \dfrac{\frac{d\sigma(W^+)}{dy} - \frac{d\sigma(W^-)}{dy}}{\frac{d\sigma(W^+)}{dy} + \frac{d\sigma(W^-)}{dy}}$

is a measure of $\dfrac{u(x_1)d(x_2) - d(x_1)u(x_2)}{u(x_1)d(x_2) + d(x_1)u(x_2)}$

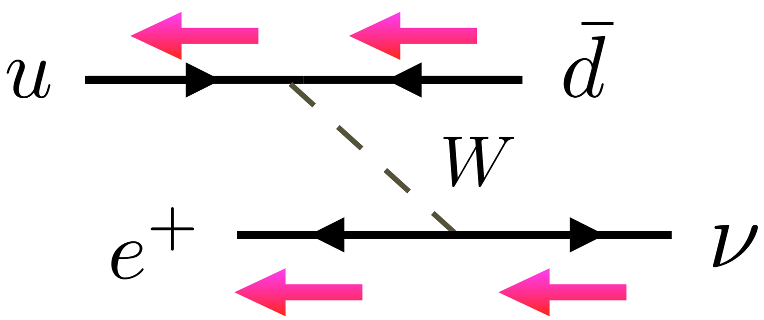


C.Anastasiou et al. (2003)

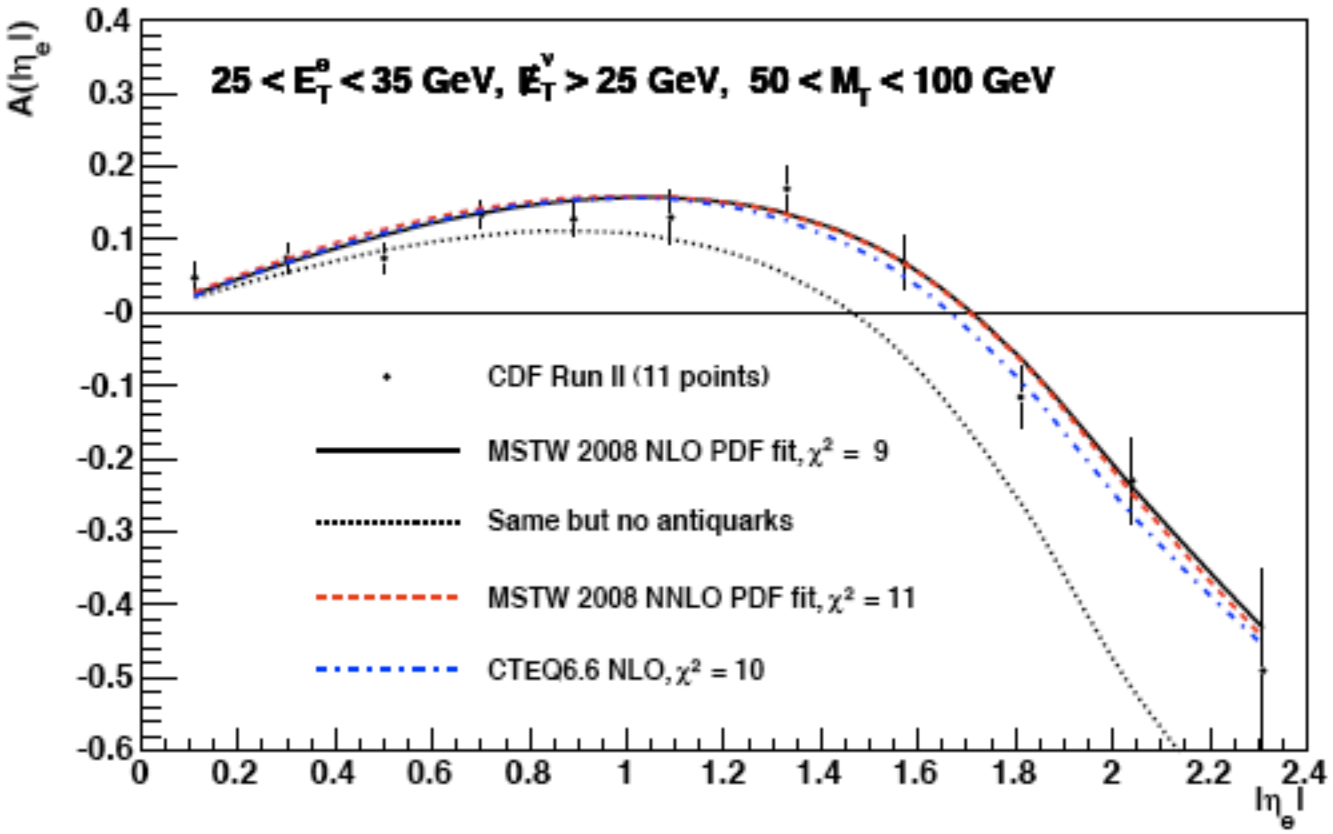
probes the relative shape of u and d quarks

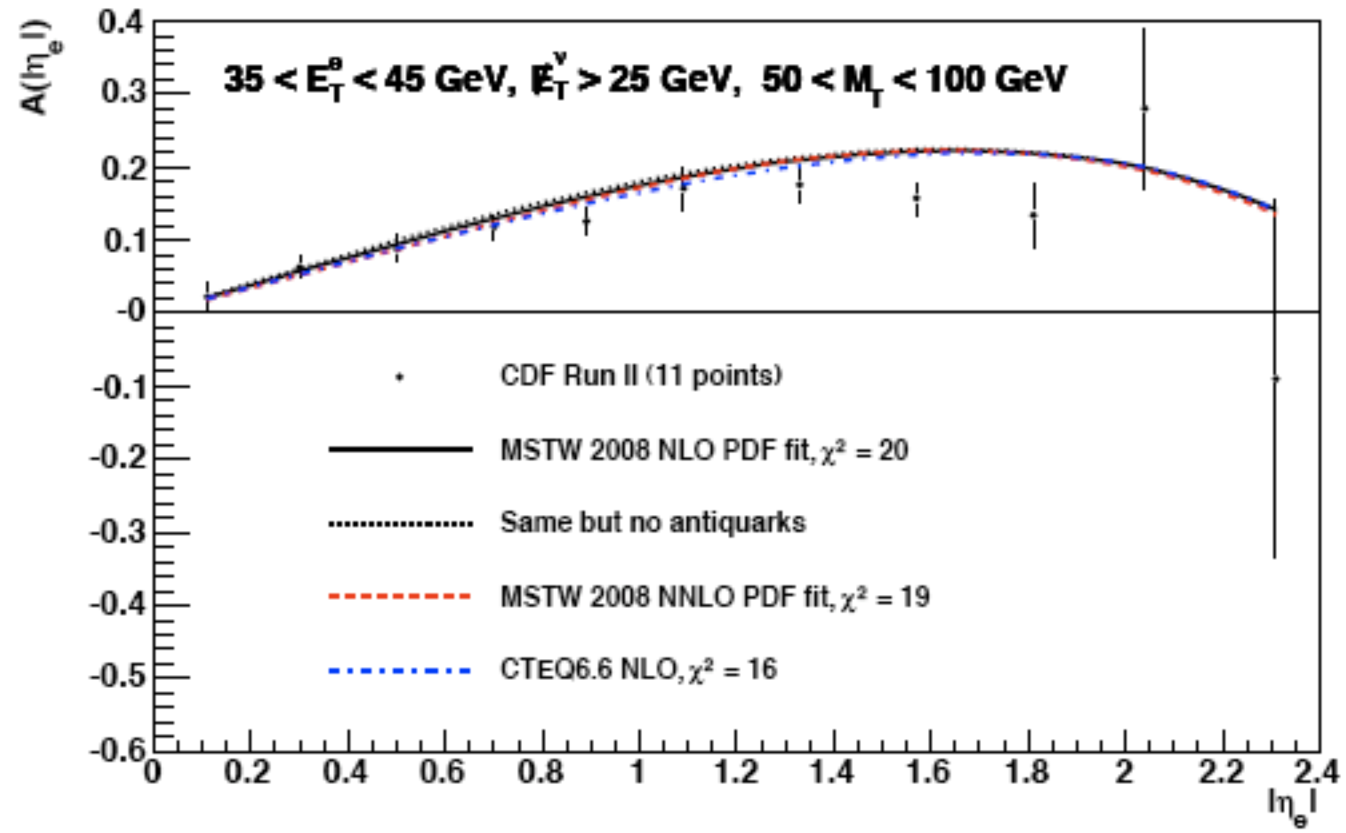
In practice $W \to l\nu$ → measure the charged lepton asymmetry

However the V-A decay of the W boson tends to dilute the effect



Angular momentum conservation: the e^+ is mainly produced in the direction of the antiquark





The effect is less evident when higher transverse energies are selected

W charge asymmetry

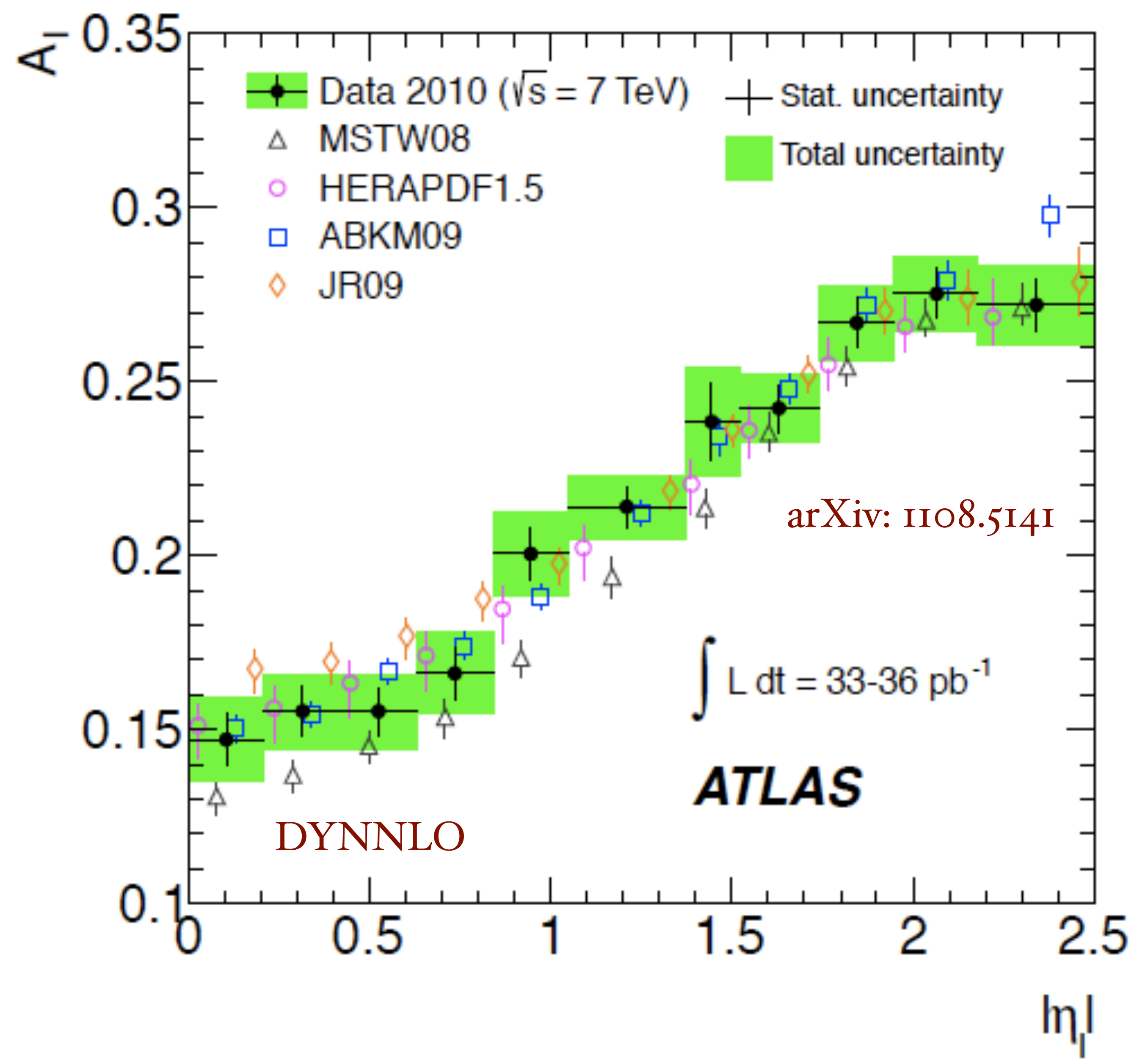
Comparison of recent ATLAS data to QCD predictions at NNLO

$p^l_T > 20$ GeV $E_T^\nu > 25$ GeV

$M_T > 40$ GeV

note that a fully exclusive calculation is needed to take cuts into account

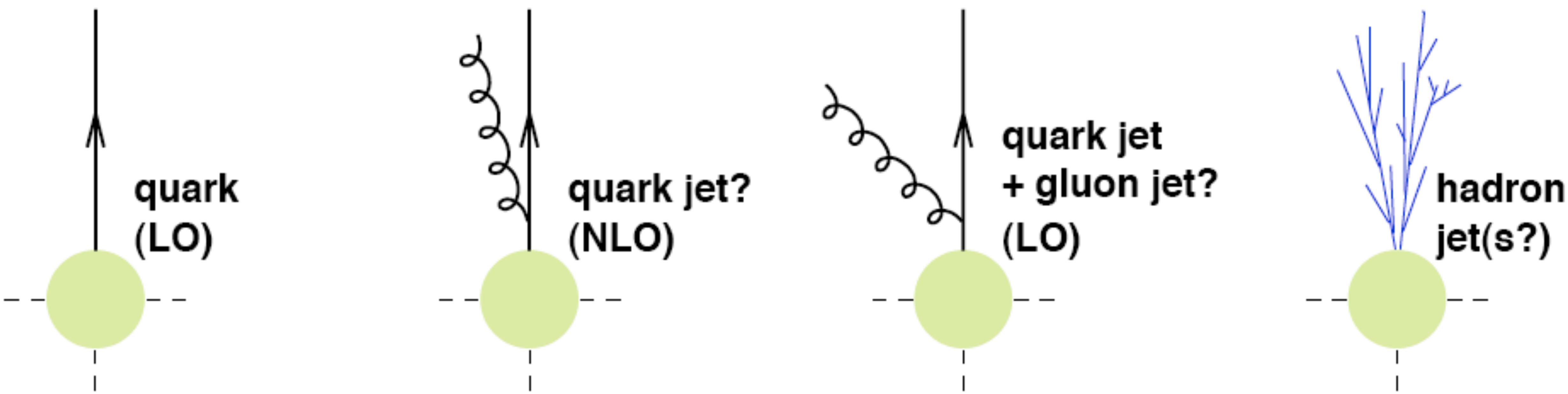
ABKM09 and HERAPDF 1.5 give best agreement with the data



Jets

It is common to discuss QCD at high-energy in terms of partons

But quarks and gluons are never really visible since, immediately after being produced they fragment and hadronize



A jet is a collimated spray of energetic hadrons and is one of the most typical manifestation of QCD at high energy

By measuring its energy and direction one can get a handle on the the original parton

How to define a jet ? A proper jet definition requires:

- a jet algorithm
- a recombination scheme

Jet algorithm: a set of rules for grouping particles into jets usually involves a set of parameters that specify how close two particles must be to belong to the same jet

Recombination scheme: indicates what momentum must be assigned to the combination of two particles (the simplest is the sum of the 4-momenta)

Snowmass accord (1990):

J.Huth et al. (1990)

Several important properties that should be met by a jet definition are:

1. Simple to implement in an experimental analysis;
2. Simple to implement in the theoretical calculation;
3. Defined at any order of perturbation theory;
4. Yields finite cross sections at any order of perturbation theory;
5. Yields a cross section that is relatively insensitive to hadronization.

Are these properties fulfilled in practice ? Not really !

Many jet algorithms at hadron colliders, some old ones have been patched, some new have been invented

Those used at hadron colliders are often IR unsafe

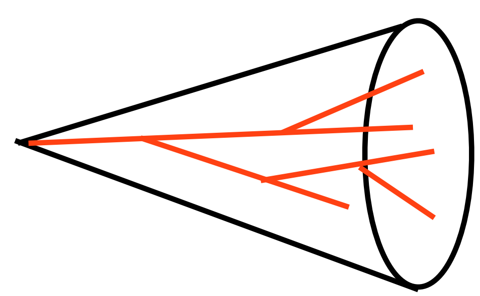
The situation is further confused by the fact that different algorithms sometimes share the same name (e.g. iterative cone)

Two broad categories:

1) cone algorithms

2) sequential recombination algorithms

1) cone algorithms
they are based on a "top-bottom" approach: rely on the idea that QCD branching and hadronization do not change the energy flow



2) sequential recombination algorithms
they are based on a "bottom-up" approach: repeatedly recombine the closest pair of particles according to some distance measure, usually related to the divergent structure of the associated QCD matrix element

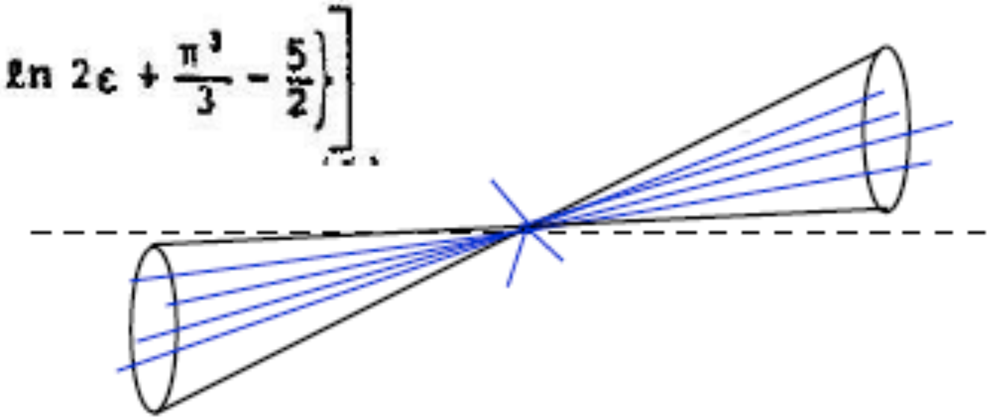
Cone algorithms

First cone algorithm dates back to original Sterman-Weinberg definition of infrared safety

G.Sterman, S.Weinberg (1977)

To study jets, we consider the partial cross section $\sigma(E,\theta,\Omega,\varepsilon,\delta)$ for e^+e^- hadron production events, in which all but a fraction $\varepsilon \ll 1$ of the total e^+e^- energy E is emitted within some pair of oppositely directed cones of half-angle $\delta \ll 1$, lying within two fixed cones of solid angle Ω (with $\pi\delta^2 \ll \Omega \ll 1$) at an angle θ to the e^+e^- beam line. We expect this to be measur-

$$\sigma(E,\theta,\Omega,\varepsilon,\delta) = (d\sigma/d\Omega)_0\,\Omega\left[1 - (g_E^2/3\pi^2)\left\{3\ln\delta + 4\ln\delta\,\ln 2\varepsilon + \frac{\pi^2}{3} - \frac{5}{2}\right\}\right]$$



The cone algorithms used today are "iterative cones" (IC) and are mostly used at hadron colliders

A seed particle i sets some initial direction, then one draws a circle around the seed of radius R in rapidity (or pseudorapidity) and azimuth, taking all j such that

$$\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2 < R^2$$

The direction of the resulting sum is then taken as a new seed and the procedure is iterated until a stable cone is found

Questions:

- How to choose the seeds ?
- What should be done when cones obtained by iterating two different seeds share some particles ?

Overlapping cones:

- First solution: *progressive removal approach*

  (often referred to as UA1-type cone algorithms)

  - Start from the particle with the largest transverse momentum
  - Once a stable cone is found, call it a jet
  - Remove all the particles contained in the cone
  - Iterate

The use of the hardest particle as seed make these algorithms *collinear* unsafe

- Second solution: *split-merge approach*

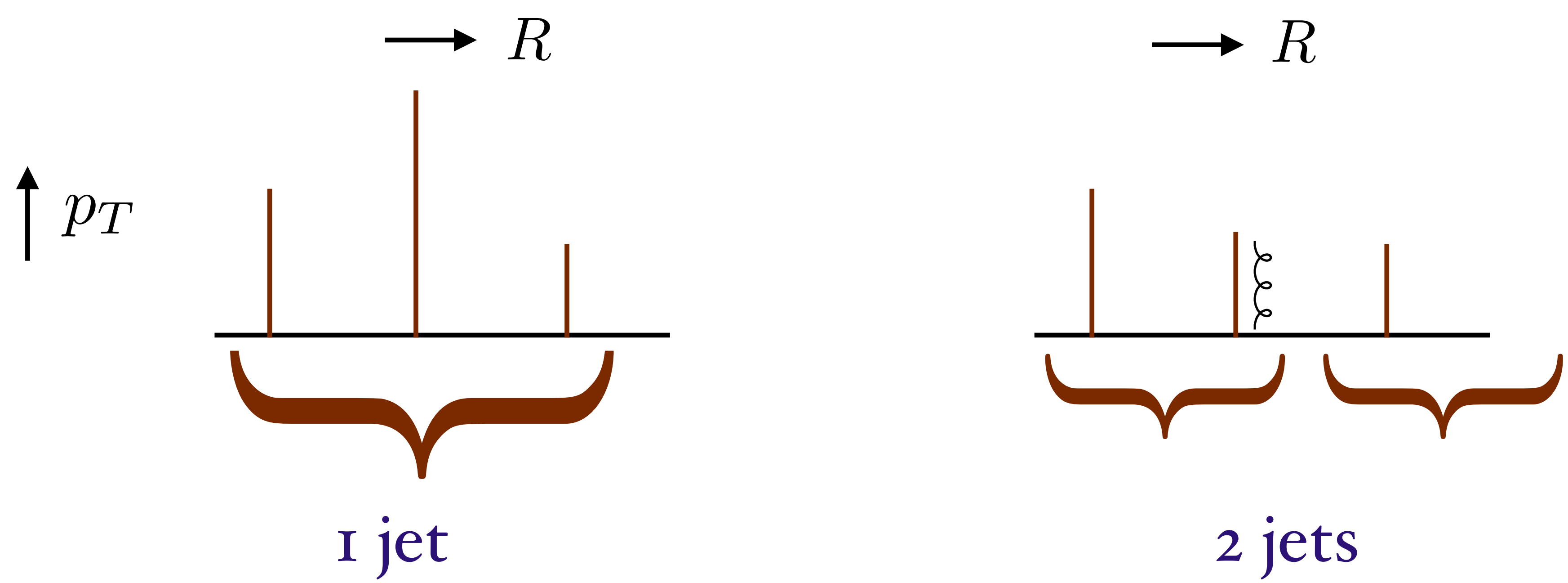
  - Find all the stable cones (protojets) starting from ALL the particles as seeds (often a threshold in p_T is assumed)

  - Run a split-merge procedure to merge a pair of cones if more than a fraction f of the softer cone's transverse momentum is shared by the harder cone

The use of seeds make these algorithms *infrared* unsafe

Infrared and collinear safety

Iterative cone algorithms with progressive removal are collinear unsafe



In the first configuration the hardest parton is the central one and if the cone is large enough we get one jet

In the second configuration the central quark has split in a collinear qg pair

➡ The number of jets should be insensitive to such a collinear splitting but now the hardest parton is the left one and we get two jets

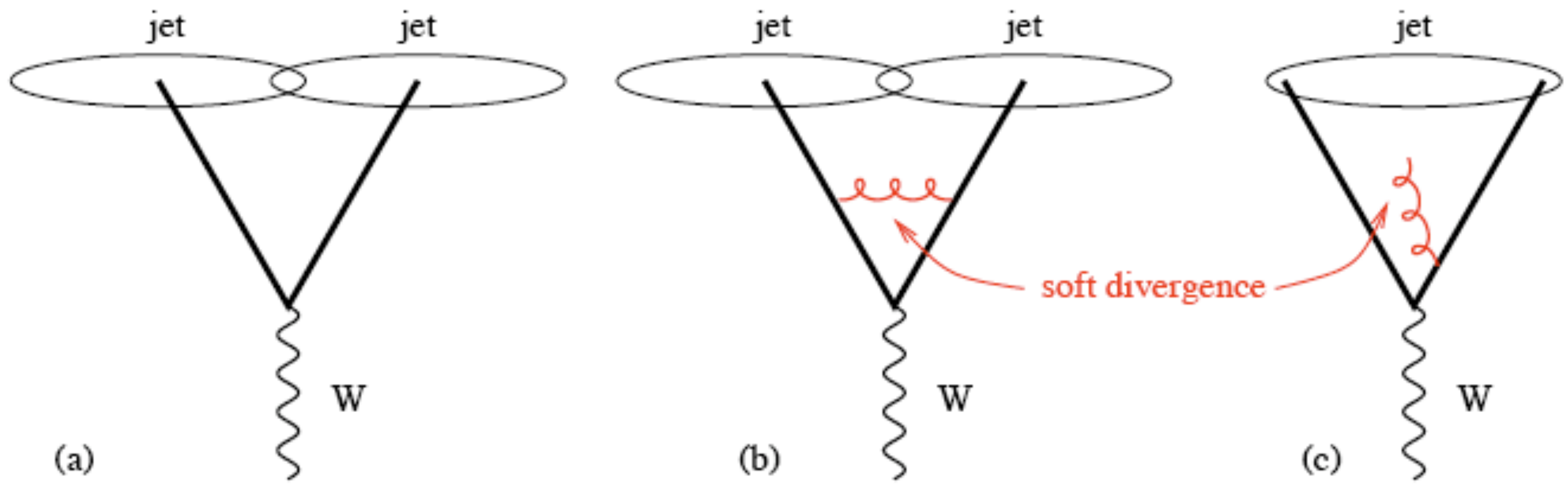
This is a serious problem for a jet-finding algorithm !

Collinear splitting are everywhere in QCD: the formal consequence is that both 1 and 2 jet cross sections are divergent in perturbation theory

In practice experimental detectors provide a regularization to the collinear unsafety, but how this happens depend on the details of tracking, electromagnetic and hadronic calorimeters

A jet cross section should not depend on the details of the detector

Iterative cone algorithms with split-merge are infrared unsafe



a) In an event with 2 hard partons both acts as seeds and give a two jet configuration

b) A virtual correction does not change the number of jets

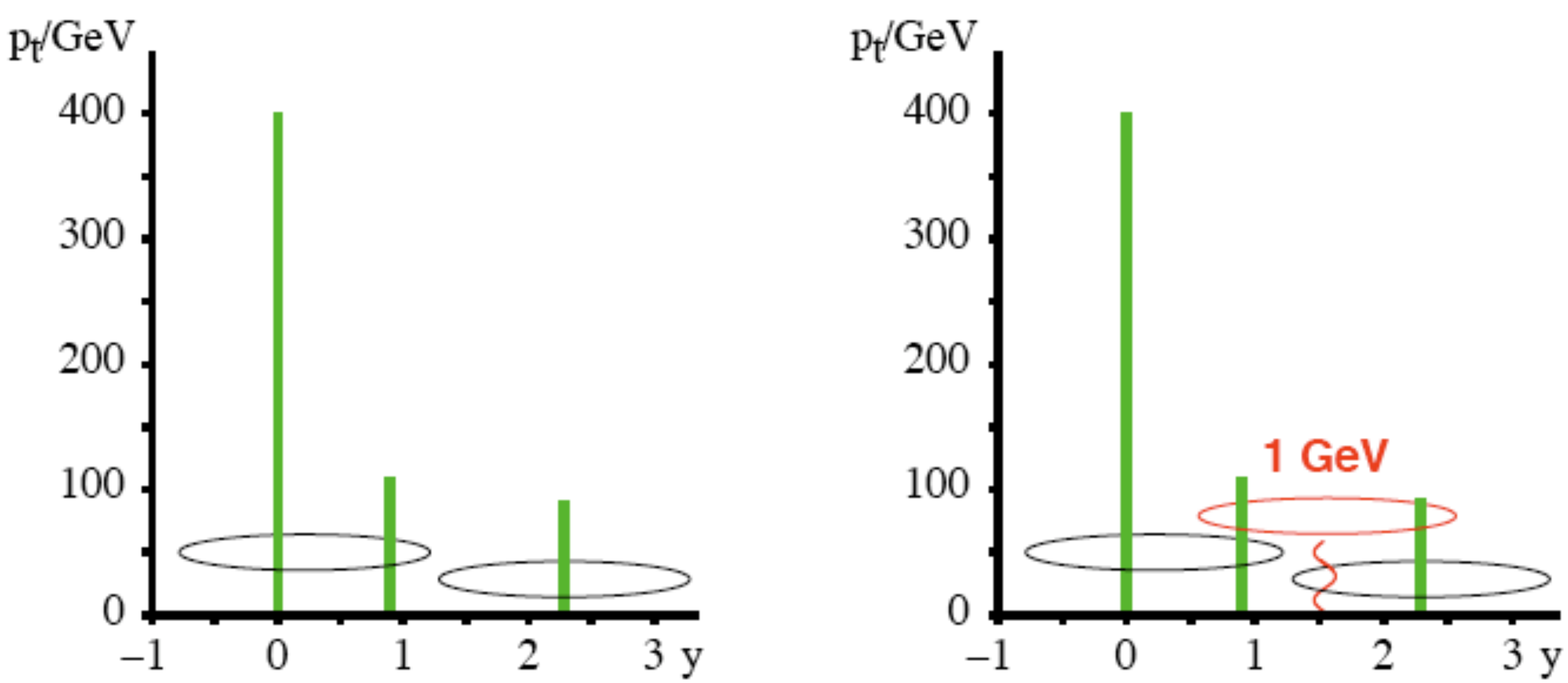
c) A soft gluon acts as a seed and may give a new stable cone
➡ a one jet configuration is found after the split-merge procedure

The algorithm is infrared unsafe and the jet cross section is divergent !

The midpoint fix:

Additionally search for new stable cones by iterating from midpoints between each pair of stable cones found in the initial seeded iteration

often presented as IR safe and widely used in Run II at the Tevatron (aka Run II cone algorithm, Improved Legacy cone) but.....



The problem reappears when three hard partons are present !

Seedless cone algorithms

Idea: find all stable cones through some exact procedure

In this way the addition of a soft particle may change the number of stable cones, but after the split merge procedure the number of stable cones does not change

Strategy:
- take all subsets of particles and establish for each of them if it corresponds to a stable cone
- calculate the total momentum, then draw a cone around it and check if all the particles within the cone are the same as in the initial subset

This may work for fixed-order calculations, with a limited number of particles but in real high-energy colliders the number of particles is large and the number of possible subsets grows like 2^N

Recently a practical seedless implementation with polynomial growth has been suggested

G.Salam, G.Soyez (2007)

Recombination schemes

Recall that a proper jet definition requires not only a jet algorithm but also a recombination scheme

The most common recombination scheme nowadays is the E-scheme, where the merging is simply done by adding the 4-momenta of the particles

A scheme that was widely used in the past is the Et-weighted scheme

$$\eta_{\rm jet} = \frac{1}{E_{T\rm jet}} \sum_i E_{Ti} \eta_i \qquad \phi_{\rm jet} = \frac{1}{E_{T\rm jet}} \sum_i E_{Ti} \phi_i$$

where $E_{T\rm jet} = \sum_i E_{Ti}$

Sequential recombination algorithms

Sequential recombination algorithms find their roots in e^+e^- experiments

- Much simpler to state than cone algorithms
- Go beyond just finding the jets: they assign a sequence to the clustering procedure that is somewhat connected to the branching at parton level

Examples:

- Jade algorithm
- k_T algorithm
- Cambridge-Achen algorithm
- anti-k_T

Jade algorithm

The first sequential recombination algorithm was introduced by the JADE collaboration in the 80's

1. For each pair ij compute the distance:

$$y_{ij} = \frac{2E_iE_j(1-\cos\theta_{ij})}{Q^2}$$

Q total energy

2. Find the minimum $y_{\min}$ of all y_{ij}

3. If $y_{\min}$ is below a threshold y_{cut} recombine i and j in a single particle (pseudojet) and repeat 1.

4. If not declare all remaining particles as jets and terminate

It depends on a single parameter y_{cut}: reducing y_{cut} resolves more jets

We may define the variable $y_{n(n+1)}$ as the value of y_{cut} at which a n jet event becomes n+1-jet like

The JADE algorithm is infrared and collinear safe: soft and collinear splitting give very small y_{ij} and thus are recombined first

However the presence of E_iE_j in the distance let two soft particles moving in opposite directions to be recombined in the same jet

→ This is against physical intuition !
We expect a jet to be limited in angular reach

Another consequence is a complication in higher order logarithmic contributions to y_{23} that cannot be resummed to all orders

The k_T algorithm in e^+e^- collisions

S.Catani et al. (1991)

The k_T algorithm in e^+e^+ collisions is identical to the JADE algorithm except for the distance measure, which is

$$y_{ij} = \frac{2\min(E_i^2, E_j^2)(1 - \cos\theta_{ij})}{Q^2}$$

In the collinear limit $\theta_{ij} \ll 1$ and the numerator becomes $(\min(E_i, E_j)\theta_{ij})^2$

It’s nothing but the squared transverse momentum of i relative to j
(i being the softer particle) → that’s why it is called k_T algorithm

In this way the distance between two soft and back to back particles is larger than that between a soft particle and a hard one close in angle

Another advantage is that the distance measure is directly related to the splitting probability in the soft and collinear limit

$$\frac{dP_{k\to ij}}{dE_i d\theta_{ij}} \sim \frac{\alpha_s}{\min(E_i, E_j)\theta_{ij}}$$

This algorithm has thus a closer relation to the structure of the divergences in QCD matrix elements, with a few nice consequences:

- the clustering sequence retains useful approximate information of the QCD branching process
- contrary to the JADE algorithm, all order resummed calculations of $y_{n(n+1)}$ are now possible

The k_T algorithm in hadron collisions

S.Catani et al. (1993)
S.D.Ellis and D.Soper (1993)

In hadronic collisions there are two difficulties to face:

- The total energy Q is not defined
- besides the divergences involving outgoing particles, there are divergences between final state and *incoming* particles

The first issue can be addressed by defining a dimensionful distance

$$d_{ij} = 2\text{min}(E_i^2, E_j^2)(1 - \cos\theta_{ij})$$

and a dimensionful jet-resolution d_{cut}

The second issue can be solved by defining an additional particle-beam distance

$$d_{iB} = 2E_i^2(1 - \cos\theta_{iB})$$

for small θ_{iB} it is just the squared transverse momentum

If there are two beams one introduces two particle beam distances

The algorithm works in the same way except for the fact that if d_{iB} is the smallest distance the particle is recombined with the beam

→ beam jets are also considered

In hadron collisions we prefer to use boost invariant quantities

→ the distance measure is defined as:

$$d_{ij} = \min(p_{Ti}^2, p_{Tj}^2)\Delta R_{ij}^2 \qquad d_{iB} = p_{Ti}^2$$

where $\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$

The algorithm defined in this way is the **exclusive k_T algorithm**

Each particle is assigned either to a jet or to a beam jet

Inclusive k_T algorithm:

$$d_{ij} = \min(p_{Ti}^2, p_{Tj}^2)\frac{\Delta R_{ij}^2}{D^2} \qquad d_{iB} = p_{Ti}^2$$

$$\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

the algorithm works as follows:

1. Compute all the distances d_{ij} and d_{iB}
2. Find the minimum.
3. If it is a d_{ij} recombine i and j and return to 1.
4. If it is a d_{iB} declare i to be a final state jet, remove it and return to 1.

There are no beam jets: each particle is assigned to a final state jet

The parameter D determines what it is called a jet:
Suppose i has no particles at a distance smaller than D:

→ d_{ij} will be larger than d_{iB} for any j harder than i

Arbitrarily soft particles can become jets in their own

→ A minimum transverse momentum for jets should be specified

The k_T algorithm has been advocated by theorists because of its good properties

Experimentalists have questioned the use of the algorithm because of its speed limit: the clustering time for N particles naively increases as N^3

The issue of speed is crucial in high-multiplicity environments like LHC or heavy-ion collisions

→ Recently the algorithm has been reformulated by using techniques borrowed from computational geometry: in this way it scales as N lnN

M.Cacciari, G.Salam (2006)

The Cambridge/Achen algorithm

It works like the inclusive k_T algorithm but using ΔR_{ij} as distance measure

It works by recombining the pair of particles with smallest ΔR_{ij} and repeating the procedure until all the clusters are separated by $\Delta R_{ij} > R$

The final objects are called jets

The clustering hierarchy is in angle rather than in transverse momentum

→ makes possible to look at the jet at different angular resolutions

G.Salam et al. (2008)

The anti-k_T algorithm

M.Cacciari, G.Salam, G.Soyez (2008)

Define a family of algorithms each characterized by an integer p

$$d_{ij} = \min(p_{Ti}^{2p}, p_{Tj}^{2p}) \frac{\Delta R_{ij}^2}{D^2} \qquad d_{iB} = p_{Ti}^{2p}$$

- p=1 k_T algorithm
- p=0 Cambridge-Aachen

What about p=-1 ? It seems a rather odd choice but...

Soft particles tend to cluster with hard ones long before they cluster among themselves

It produces regular (circular) jets

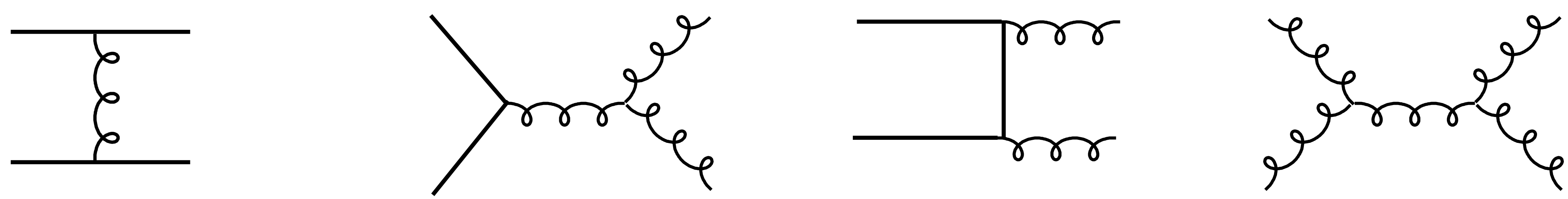
A sequential recombination algorithm is the perfect cone algorithm !

Now the default for ATLAS and CMS experiments

Jet production at hadron colliders

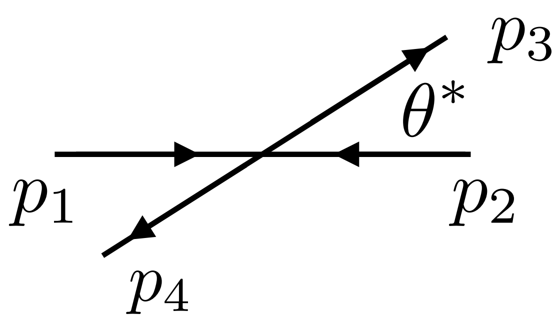
Two-jet events are produced in QCD when the incoming partons produce two high transverse momentum outgoing partons

Some of the diagrams:



$$d\hat{\sigma} = \frac{1}{2\hat{s}} \sum |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) \frac{dp_3^3}{2E_3(2\pi)^3} \frac{dp_4^3}{2E_4(2\pi)^3}$$

Kinematics:



$$p_1 = \frac{\sqrt{\hat{s}}}{2}(1,0,0,1) \qquad p_2 = \frac{\sqrt{\hat{s}}}{2}(1,0,0,-1)$$

In the CM frame

$$p_3 = \frac{\sqrt{\hat{s}}}{2}(1, \sin\theta^* \sin\phi, \sin\theta^* \cos\phi, \cos\theta^*)$$

$$p_4 = \frac{\sqrt{\hat{s}}}{2}(1, -\sin\theta^* \sin\phi, -\sin\theta^* \cos\phi, -\cos\theta^*)$$

$$\hat{s} = (p_1 + p_2)^2 \qquad \hat{t} = (p_1 - p_3)^2 = -\frac{\hat{s}}{2}(1 - \cos\theta^*)$$

$$\hat{u} = (p_2 - p_3)^2 = -\frac{\hat{s}}{2}(1 + \cos\theta^*)$$

$$y_3^{\mathrm{CM}} = -y_4^{\mathrm{CM}} \equiv y^* \qquad y^* = (y_3 - y_4)/2 \qquad \cos\theta^* = \tanh y^*$$

➡ A measure of the rapidity difference of the two jets gives the scattering angle in the centre-of-mass frame

Define the rapidity of the two parton system $\bar{y}$

$$x_1 = x_T \cosh y^* e^{\bar{y}}$$

where $x_T = 2p_T/\sqrt{\hat{s}}$

$$x_2 = x_T \cosh y^* e^{-\bar{y}}$$

The invariant mass of the two-jet system can be written as

$$m_{JJ}^2 = \hat{s} = 4p_T^2 \cosh^2 y^*$$

The partonic inclusive jet cross section can be obtained by integrating over the momentum of one of the jets

$$d\hat{\sigma} = \frac{1}{2\hat{s}} \sum |\mathcal{M}|^2 \frac{1}{4(2\pi)^2} \frac{d^3p_3}{E_3 E_4} \delta(E_1 + E_2 - E_3 - E_4) = \frac{1}{2\hat{s}} \sum |\mathcal{M}|^2 \frac{1}{8\pi^2} \frac{d^3p_3}{E_3} \delta(\hat{s} + \hat{t} + \hat{u})$$

The corresponding hadronic cross section is

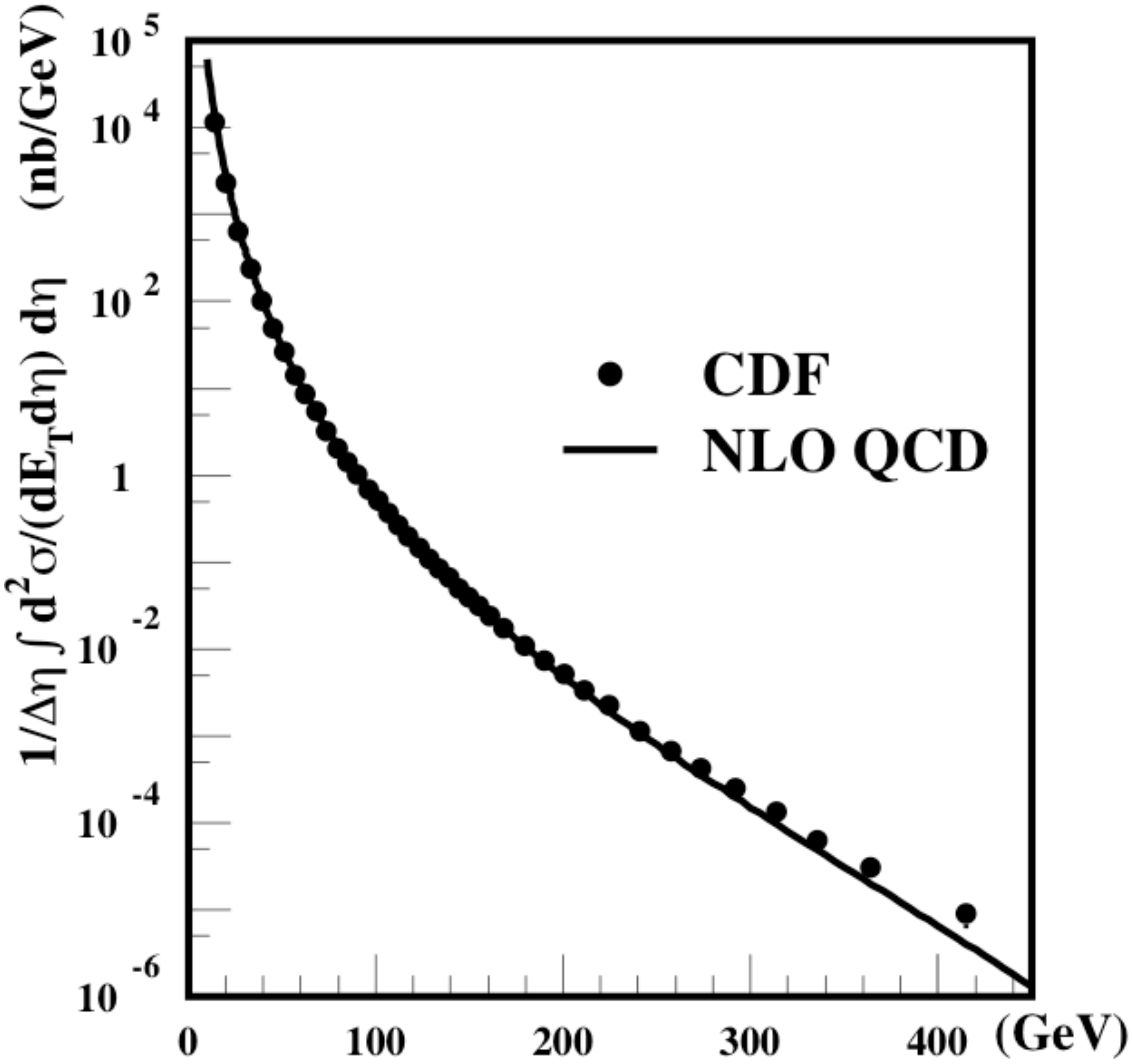
$$\frac{d\sigma}{d^2p_T dy} = \frac{1}{16\pi^2 s} \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} f_i(x_1, \mu_F^2) f_j(x_2, \mu_F^2) \sum_{i,j,k,l} \sum |\mathcal{M}(ij \to kl)|^2 \delta(\hat{s} + \hat{t} + \hat{u})$$

Comparison with data

hep-ex/9601008

The measurement of the CDF collaboration at Run I at the Tevatron was historically very important

Spectacular agreement of the data with NLO QCD over nine order of magnitude !



Comparison with data

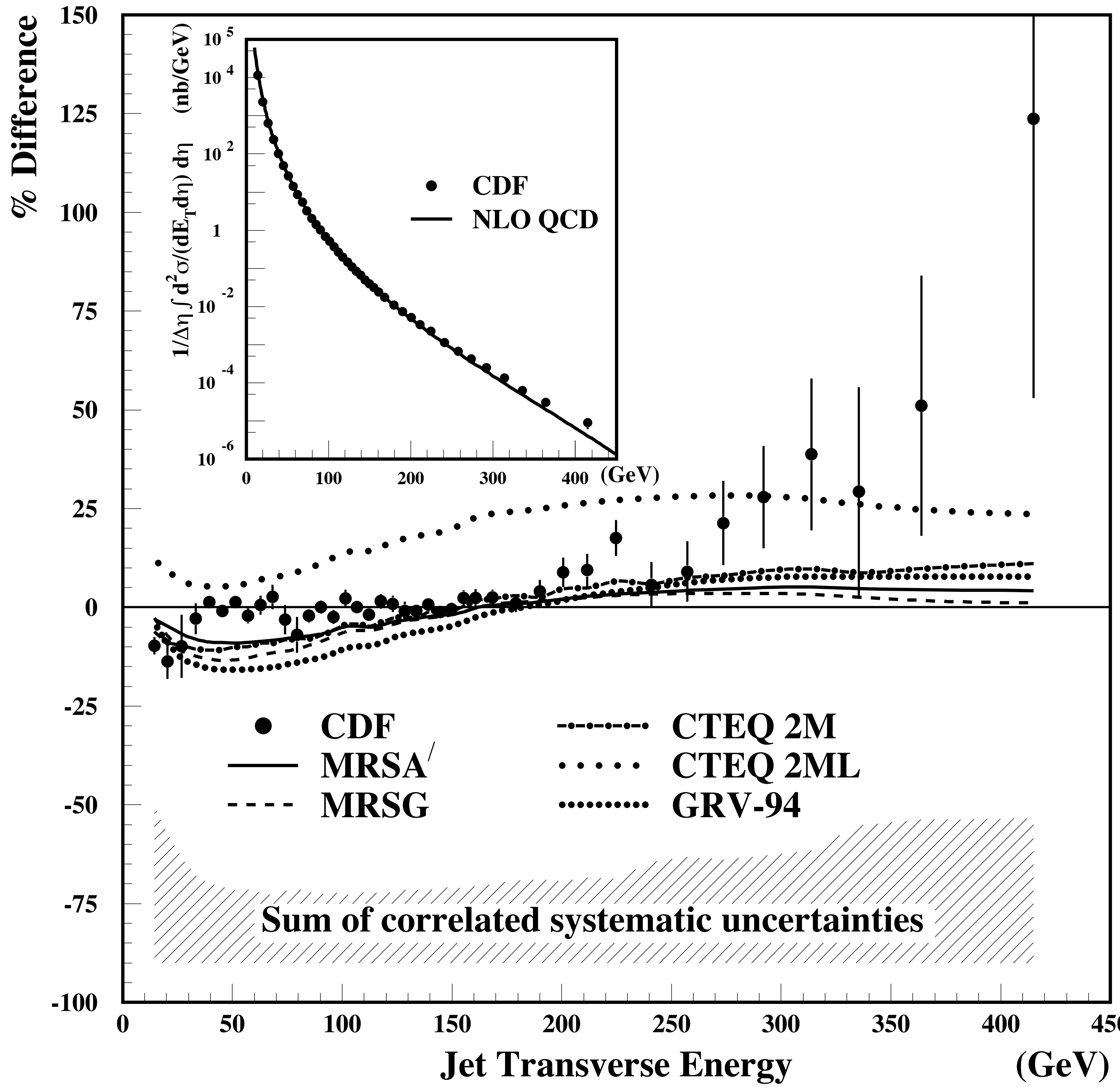
hep-ex/9601008

The measurement of the CDF collaboration at Run I at the Tevatron was historically very important

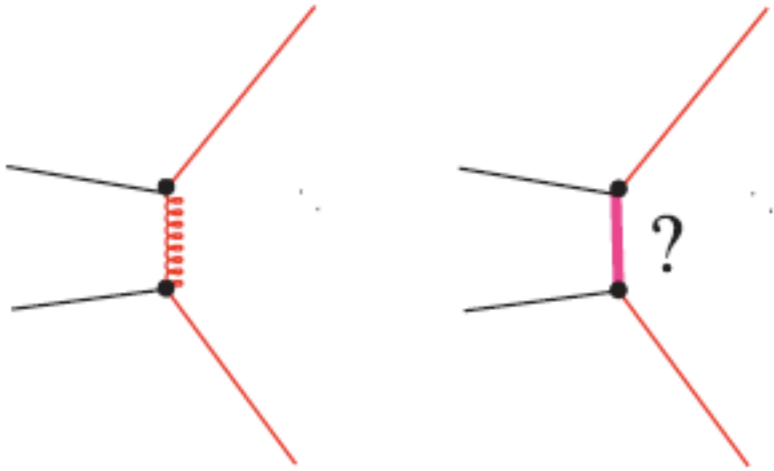
Spectacular agreement of the data with NLO QCD over nine order of magnitude !

At high transverse energies the data disagree with the theoretical prediction

Many new physics interpretations were proposed



Quark compositeness typically produces four fermion contact interactions due to the exchange of some heavy new particle of mass M



$$\Delta\mathcal{L} = \frac{f^2}{M^2}\bar{\psi}\gamma^\mu\psi\,\bar{\psi}\gamma_\mu\psi$$

→ should lead to
$$\frac{\text{Data} - \text{Theory}}{\text{Theory}} \sim f^2\frac{E_T^2}{M^2}$$

Such interactions however would lead to observable effects in the jet angular distributions

Consider the partonic cross section $\dfrac{d\hat{\sigma}_{ij}}{d\cos\theta^*}$

The dominant channels in $p\bar{p}$ are $gq \to gq\,,\; gg \to gg\;,\; q\bar{q} \to q\bar{q}$

All these channels have the familiar Rutherford singularity

$$\frac{d\hat{\sigma}}{d\cos\theta^*} \sim \frac{1}{\sin^4(\theta^*/2)}$$

Due to the exchange of a massless boson in the t channel

➡ Define the variable $\chi = \dfrac{1+\cos\theta^*}{1-\cos\theta^*}$

Note that the variable χ in LO QCD is related to the rapidity difference of the two jets through $\chi=\exp\{y_3-y_4\}$

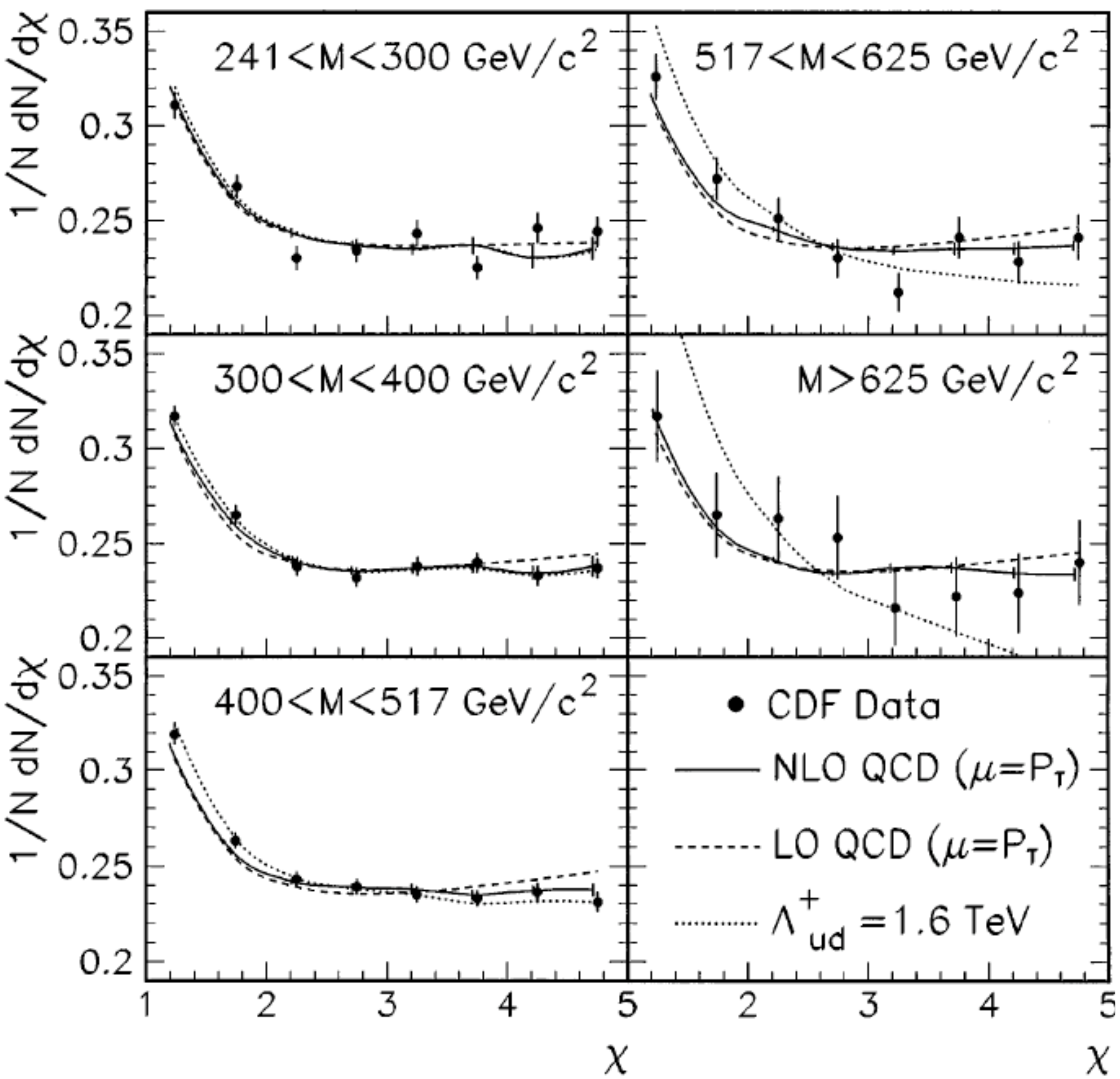
The change of variable leads to $\dfrac{d\hat{\sigma}}{d\chi} \sim \text{constant}$ for the QCD prediction

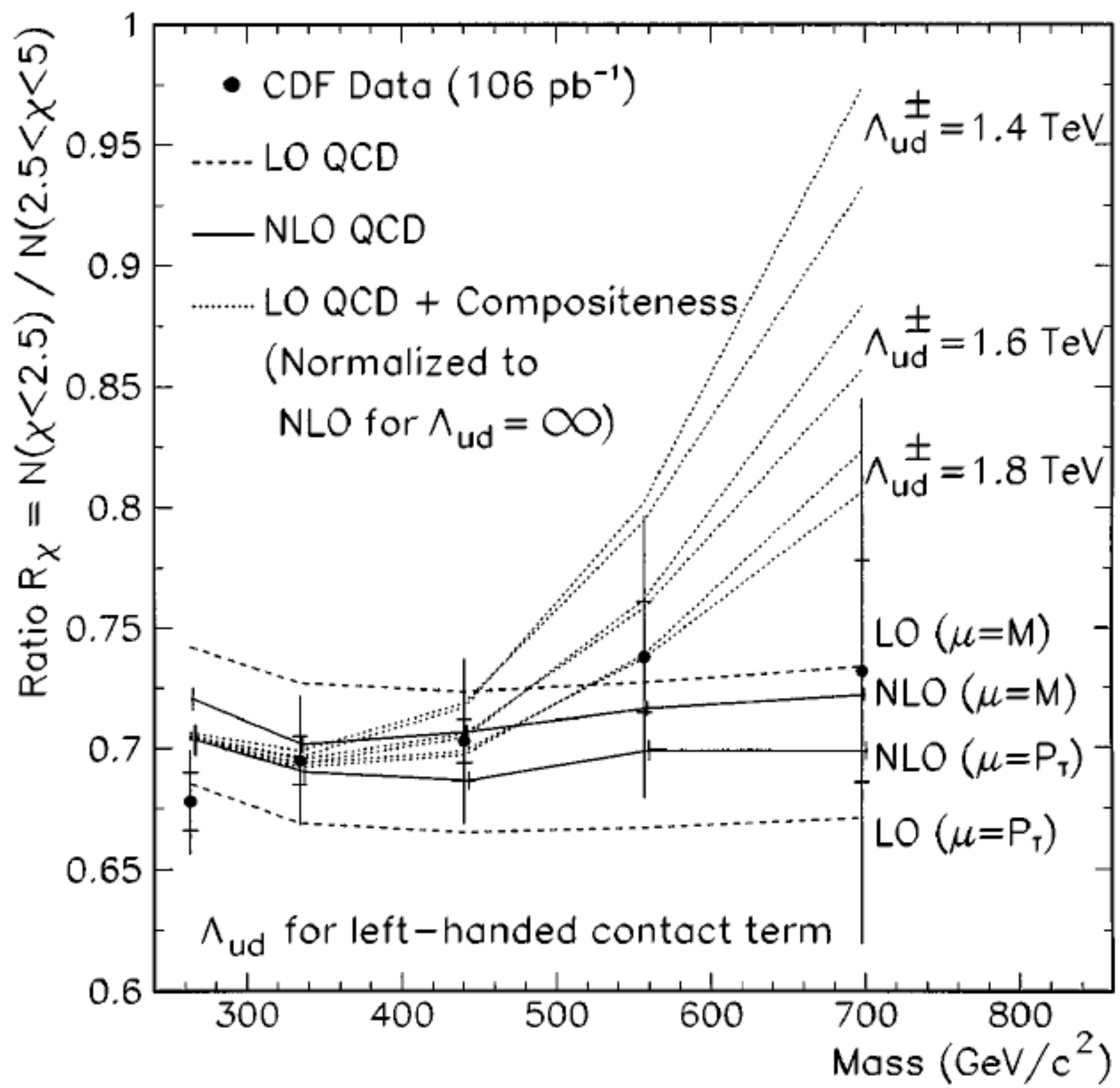
On the contrary, the exchange of a scalar particle leads to

$$\frac{d\hat{\sigma}}{d\cos\theta^*} \sim \text{constant} \qquad \text{and thus} \qquad \frac{d\hat{\sigma}}{d\chi} \sim \frac{1}{(1+\chi)^2}$$

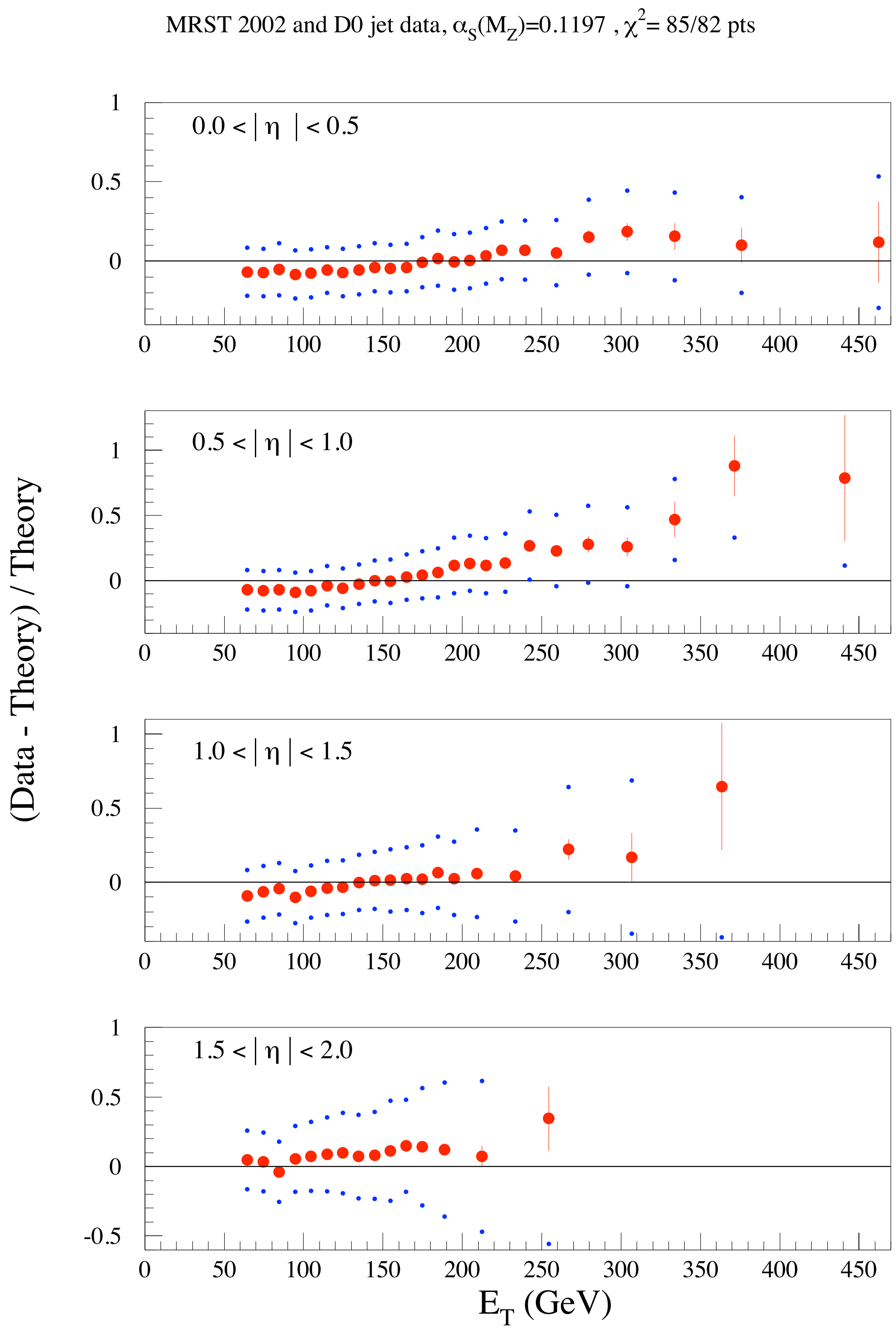
This distribution has been measured by the CDF collaboration

The results do not show significant deviations in the angular distribution and exclude that the excess at high E_T is due to new contact interactions arising in compositeness scenarios





Later it was understood that the excess could be reabsorbed by a suitable modifications of parton distribution functions



Jet production at large transverse energies is in fact unfortunately the only constraint on the gluon distribution at large x

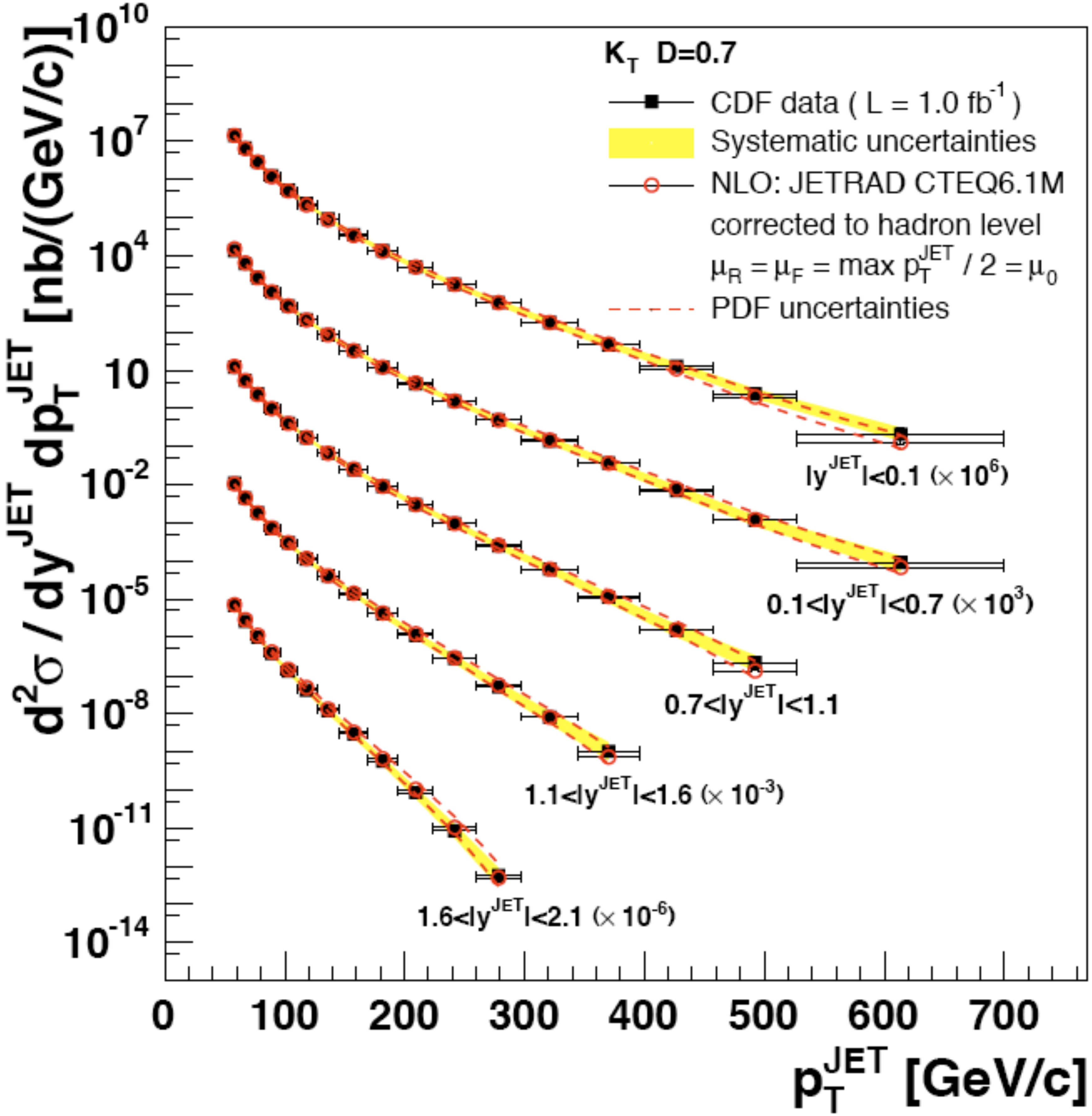
Strong interplay with new physics effects

Inclusive jet cross section measured by CDF Run II with the k_T algorithm

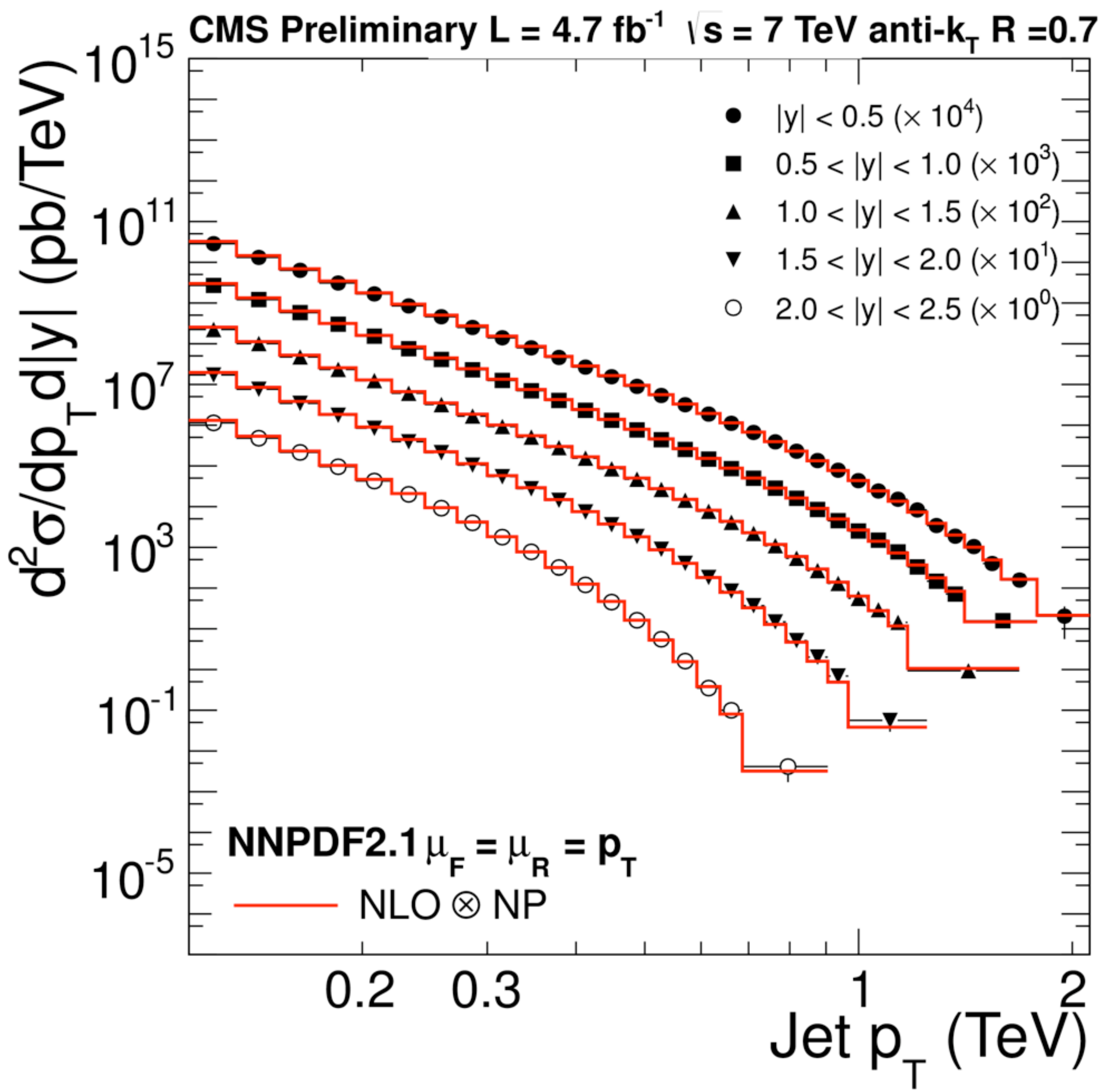
hep-ex/0701051

Good agreement with NLO QCD

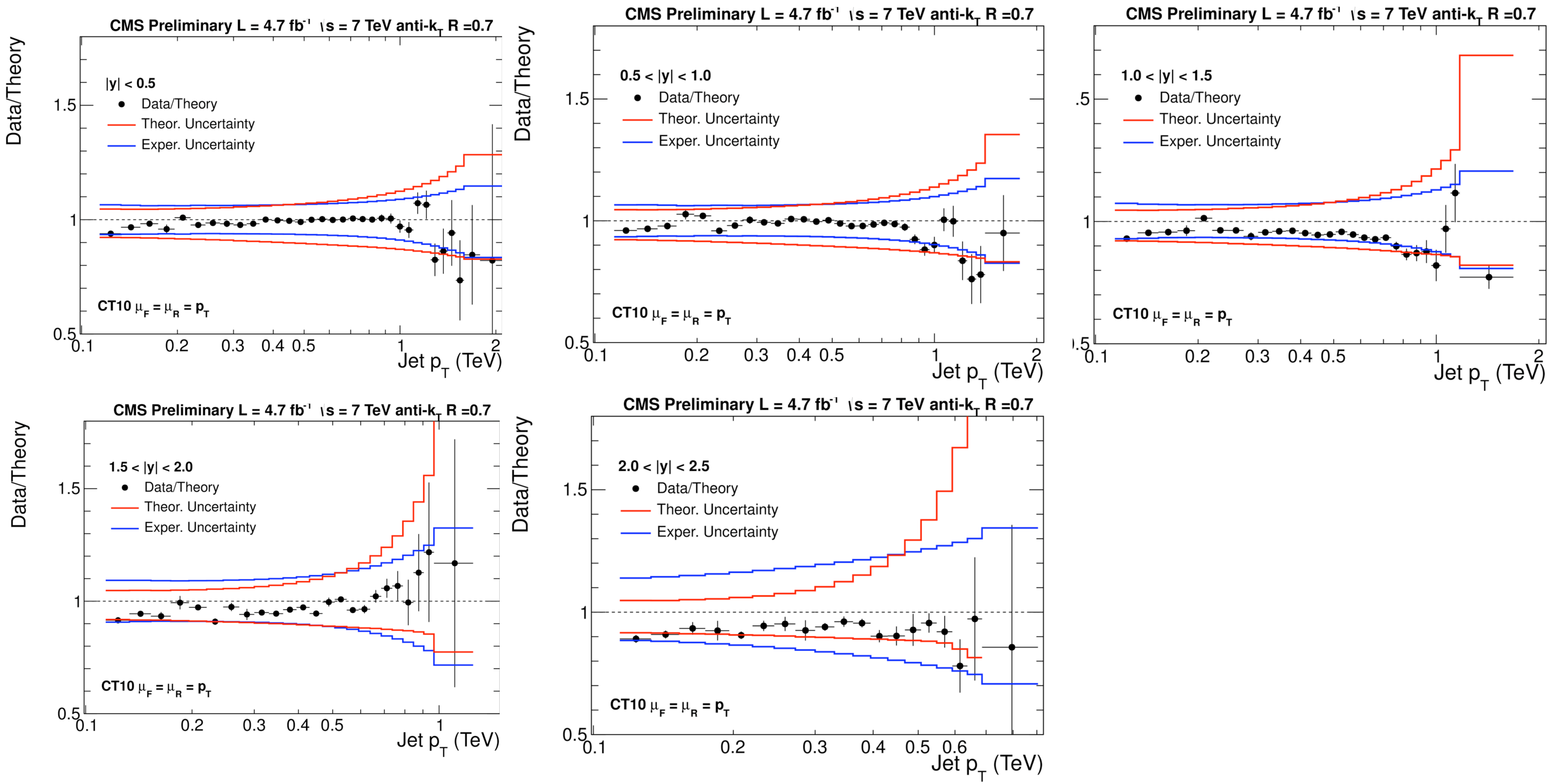
UE corrections similar to those found with cone



These studies now continued at the LHC



Good agreement with data up to p_T of order 2 TeV !


CMS Preliminary L = 4.7 fb-1 \s = 7 TeV anti-kT R =0.7
|y| < 0.5
Data/Theory
Theor. Uncertainty
Exper. Uncertainty
CT10 μF = μR = pT
Data/Theory
Jet pT (TeV)
CMS Preliminary L = 4.7 fb-1 \s = 7 TeV anti-kT R =0.7
0.5 < |y| < 1.0
Data/Theory
Theor. Uncertainty
Exper. Uncertainty
CT10 μF = μR = pT
Data/Theory
Jet pT (TeV)
CMS Preliminary L = 4.7 fb-1 \s = 7 TeV anti-kT R =0.7
1.0 < |y| < 1.5
Data/Theory
Theor. Uncertainty
Exper. Uncertainty
CT10 μF = μR = pT
Jet pT (TeV)
CMS Preliminary L = 4.7 fb-1 \s = 7 TeV anti-kT R =0.7
1.5 < |y| < 2.0
Data/Theory
Theor. Uncertainty
Exper. Uncertainty
CT10 μF = μR = pT
Data/Theory
Jet pT (TeV)
CMS Preliminary L = 4.7 fb-1 \s = 7 TeV anti-kT R =0.7
2.0 < |y| < 2.5
Data/Theory
Theor. Uncertainty
Exper. Uncertainty
CT10 μF = μR = pT
Data/Theory
Jet pT (TeV)

Parton distributions

Determined by global fits to different data sets

Standard procedure:

- Parametrize at input scale $Q_0 = 1 - 4\ \mathrm{GeV}$

$$xf(x, Q_0^2) = Ax^{\alpha}(1-x)^{\beta}(1 + \epsilon\sqrt{x} + \gamma x + ...)$$

- Impose momentum sum rule: $\sum_a \int_0^1 dx x f(x, Q_0^2) = 1$
- Evolve to desired Q^2 and compute physical observables
- Then fit to data to obtain the parameters

Main groups: MRST (now MSTW), CTEQ

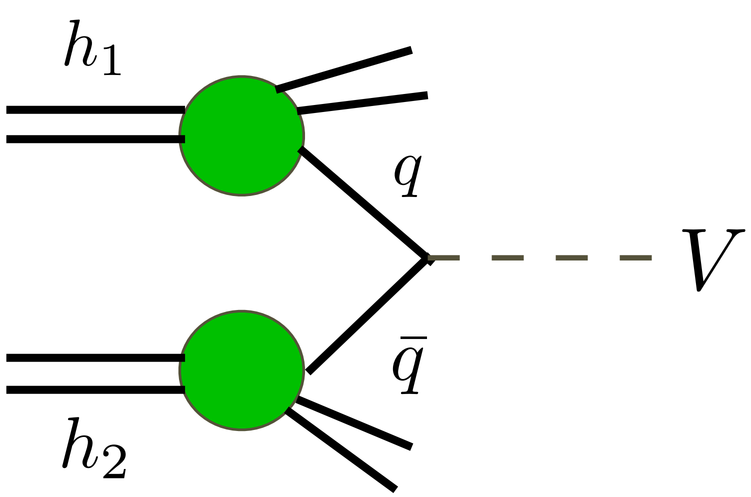
Now also: Alekhin, Delgado-Reya, HERA, NNPDF..

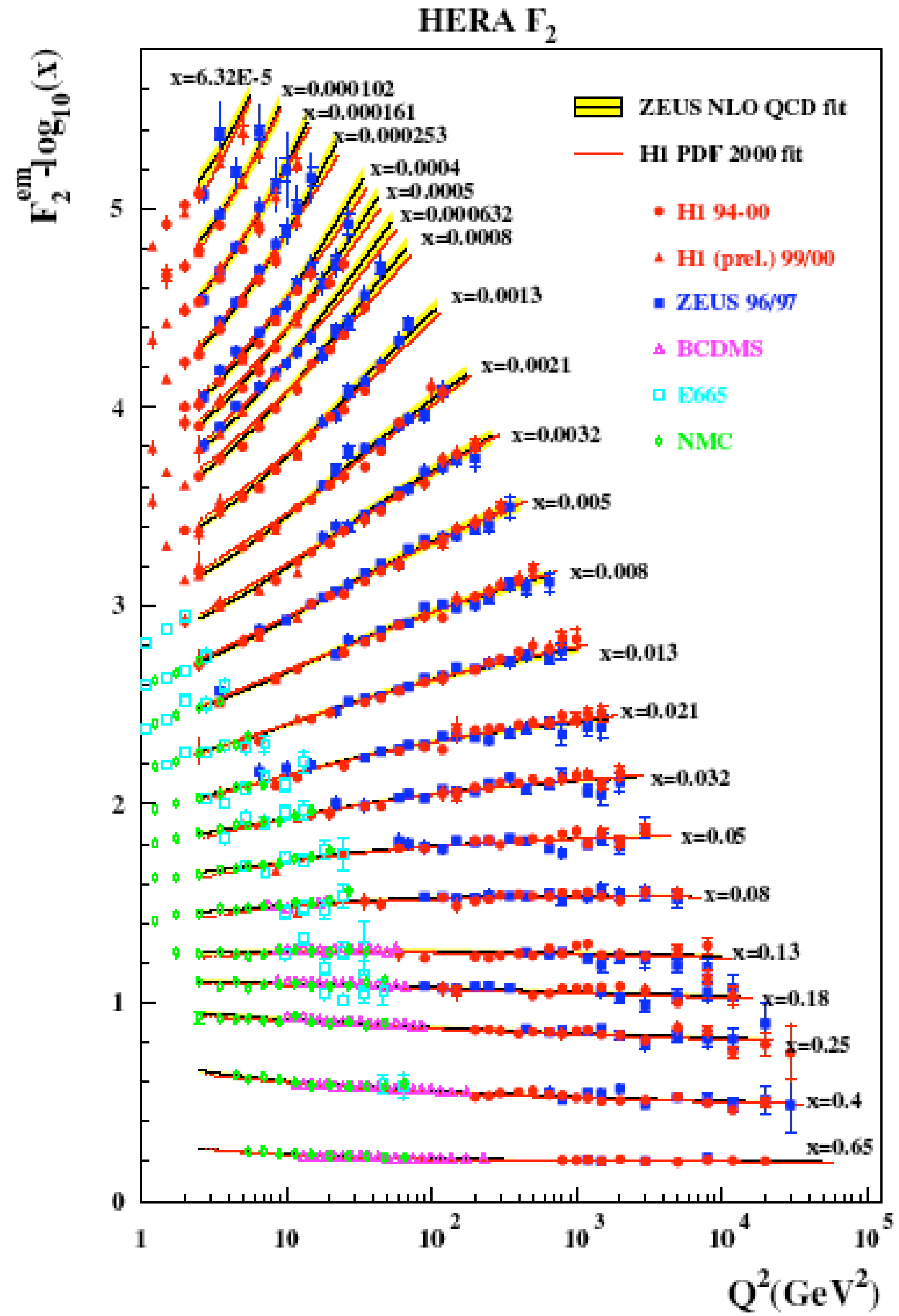
Typical processes:

- DIS:

  Fixed target: valence quark densities
  HERA: f_g, f_{sea} at small x

- Drell-Yan → quark densities



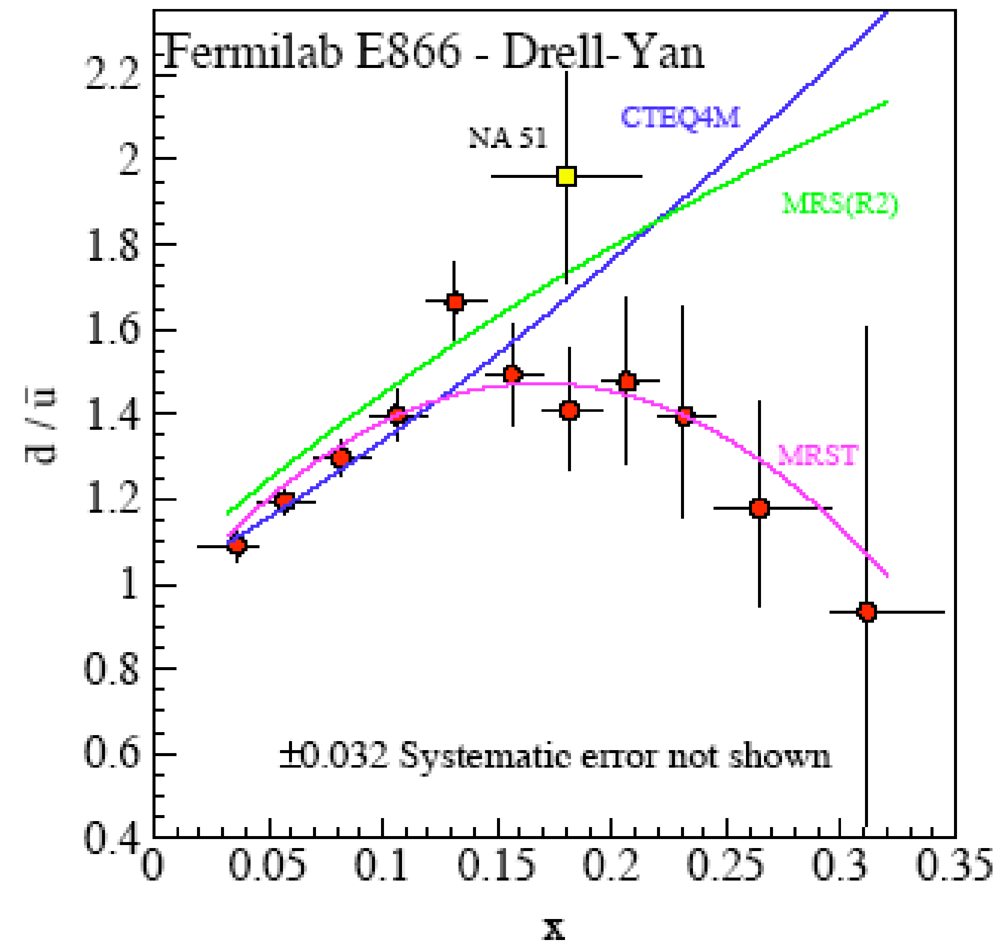


pp collisions: sensitive to antiquarks and sea densities

$p\bar{p}$ collisions: sensitive to flavour asymmetries of valence quarks

$\bar{d}(x)/\bar{u}(x)$ from FNAL E866/Nusea

800 GeV $p+p$ and $p+d \to \mu^+\mu^-$



Obtain neutron pdfs from isospin symmetry:

$$u \leftrightarrow d$$
$$\bar{u} \leftrightarrow \bar{d}$$

$$\sigma^{pp} \sim \frac{4}{9}u(x_1)\bar{u}(x_2) + \frac{1}{9}d(x_1)\bar{d}(x_2)$$

$x_1 \gg x_2$

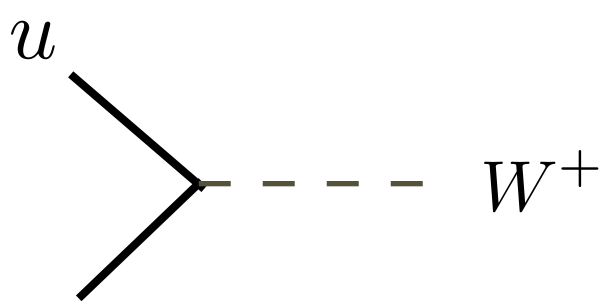
$$\sigma^{pn} \sim \frac{4}{9}u(x_1)\bar{d}(x_2) + \frac{1}{9}d(x_1)\bar{u}(x_2)$$

Assuming $\sigma^{pd} \sim \sigma^{pp} + \sigma^{pn}$

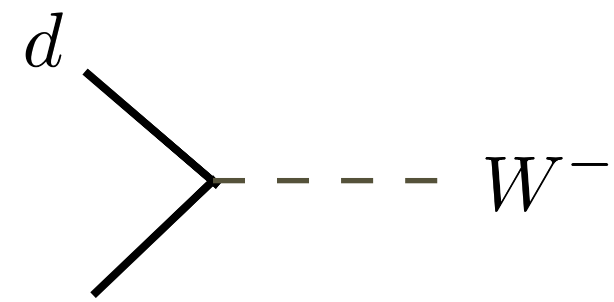
and using $d(x) \ll 4u(x)$

$$\left.\frac{\sigma^{pd}}{2\sigma^{pp}}\right|_{x_1 \gg x_2} \sim \frac{1}{2}\frac{1+\frac{1}{4}\frac{d(x_1)}{u(x_1)}}{1+\frac{1}{4}\frac{d(x_1)\bar{d}(x_2)}{u(x_1)\bar{u}(x_2)}}\left(1+\frac{\bar{d}(x_2)}{\bar{u}(x_2)}\right) \quad \Rightarrow \quad \frac{\sigma^{pd}}{2\sigma^{pp}} \sim \frac{1}{2}\left(1+\frac{\bar{d}_2}{\bar{u}_2}\right)$$

W asymmetry in $p\bar{p}$ collisions



$\bar{d} = d$ in the proton



$\bar{u} = u$ in the proton

If u in the proton is faster than d
$(u(x) > d(x))$

➡ $W^+(W^-)$ produced mainly in $p\ (\bar{p})$ direction

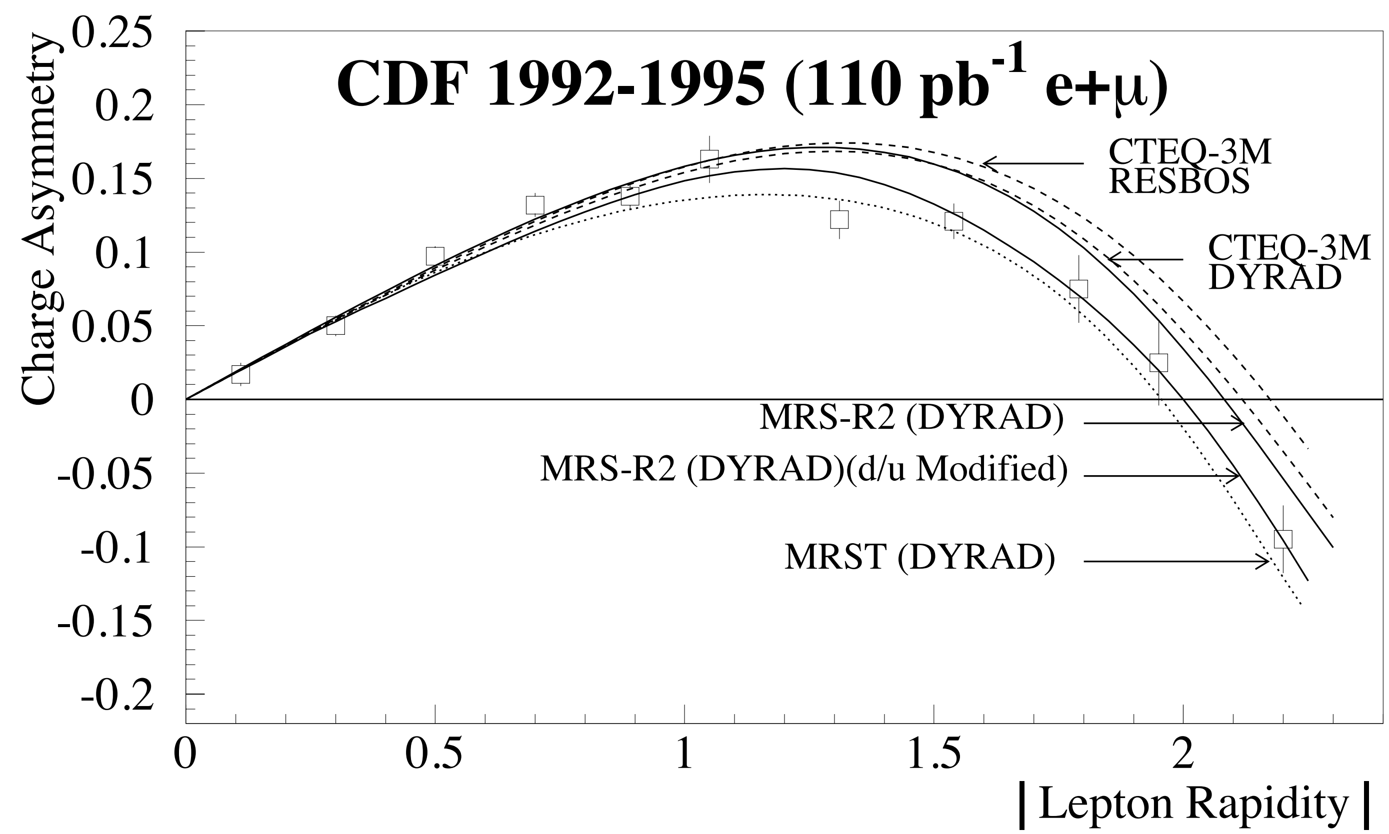
The W asymmetry $A(y) = \dfrac{\frac{d\sigma(W^+)}{dy} - \frac{d\sigma(W^-)}{dy}}{\frac{d\sigma(W^+)}{dy} + \frac{d\sigma(W^-)}{dy}}$

is a measure of

$$\frac{u(x_1)d(x_2) - d(x_1)u(x_2)}{u(x_1)d(x_2) + d(x_1)u(x_2)}$$

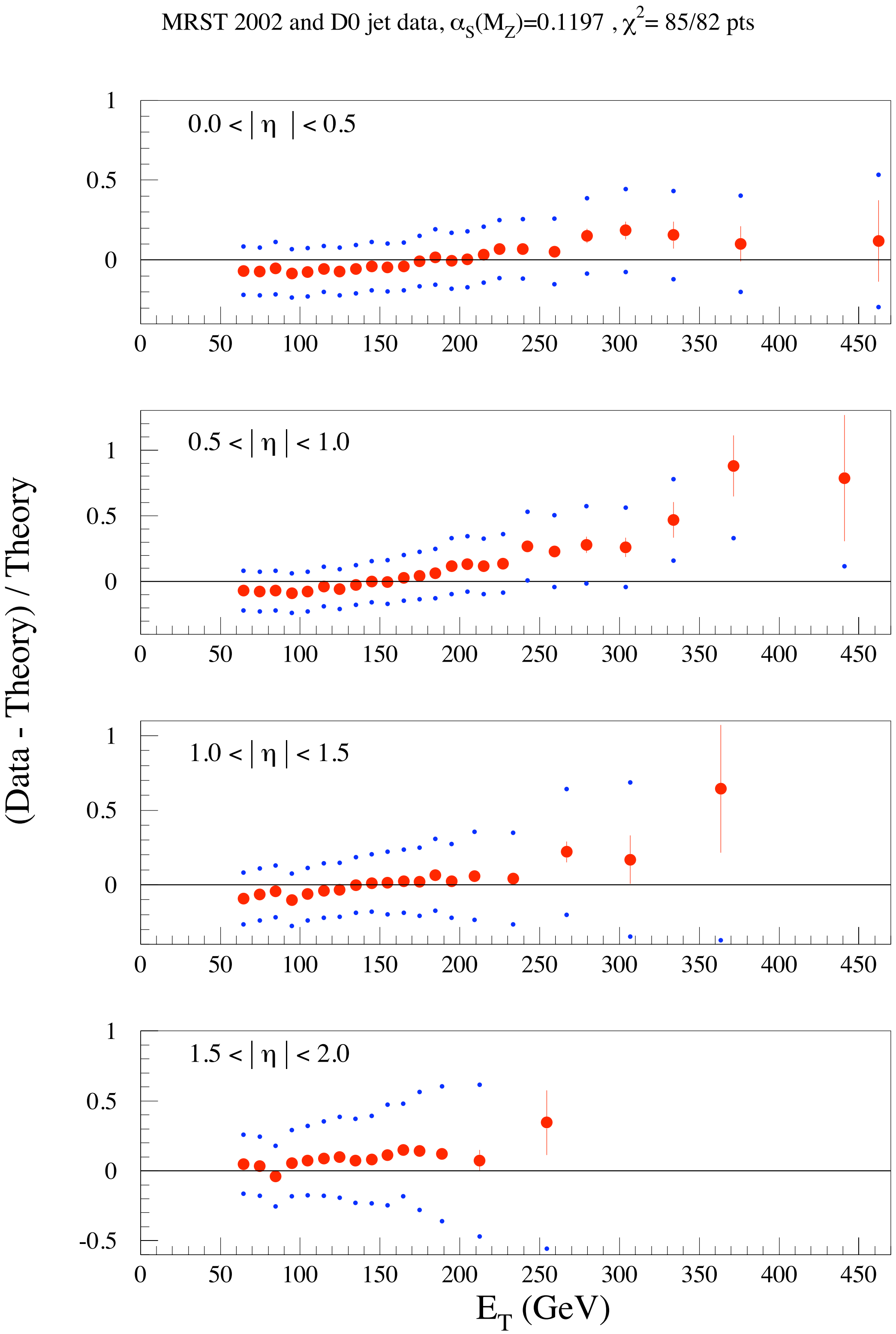
In practice $W \to l\nu$

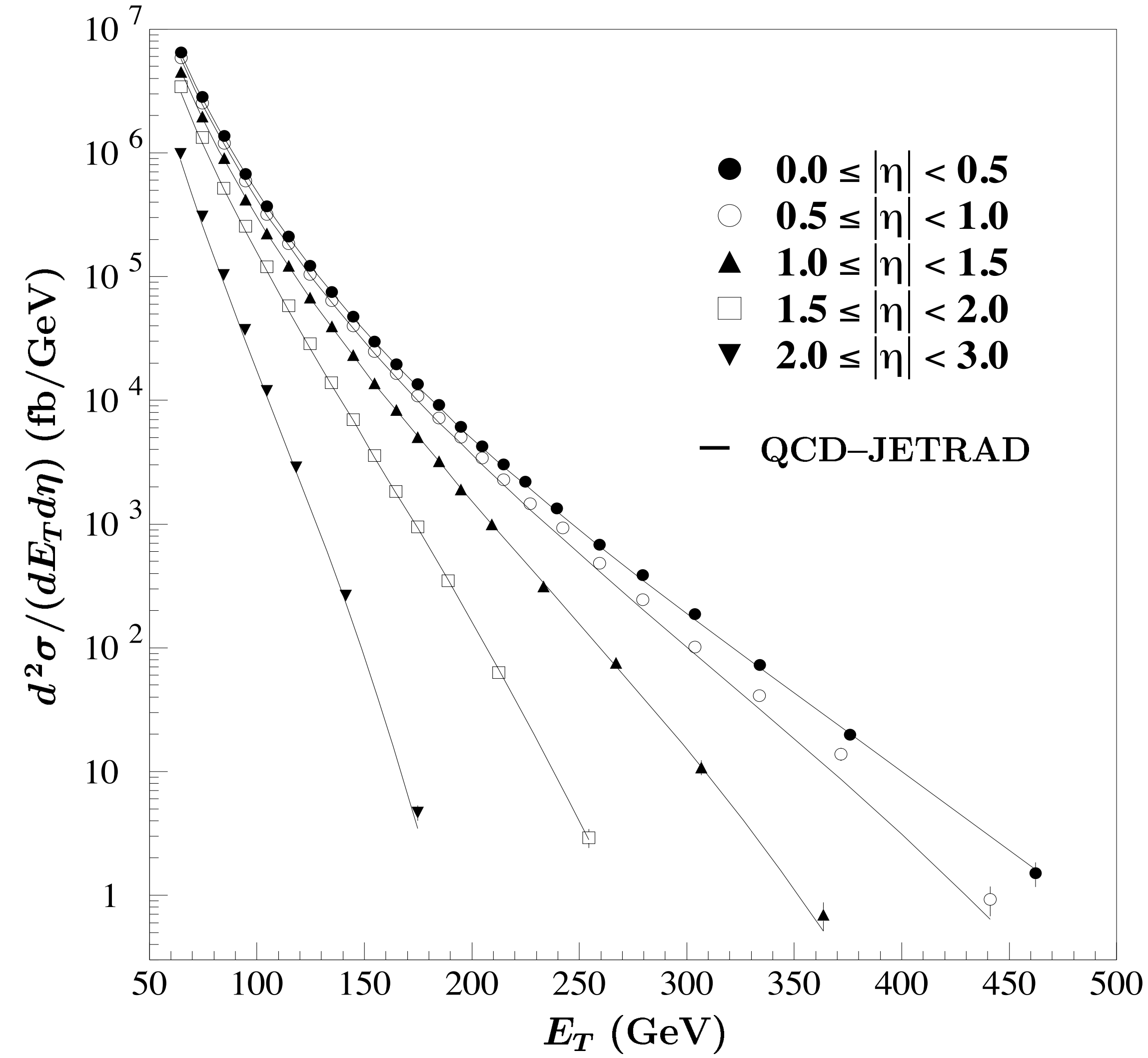
➡ measure the charged lepton asymmetry



- Jets at Tevatron

➡ f_g at large x

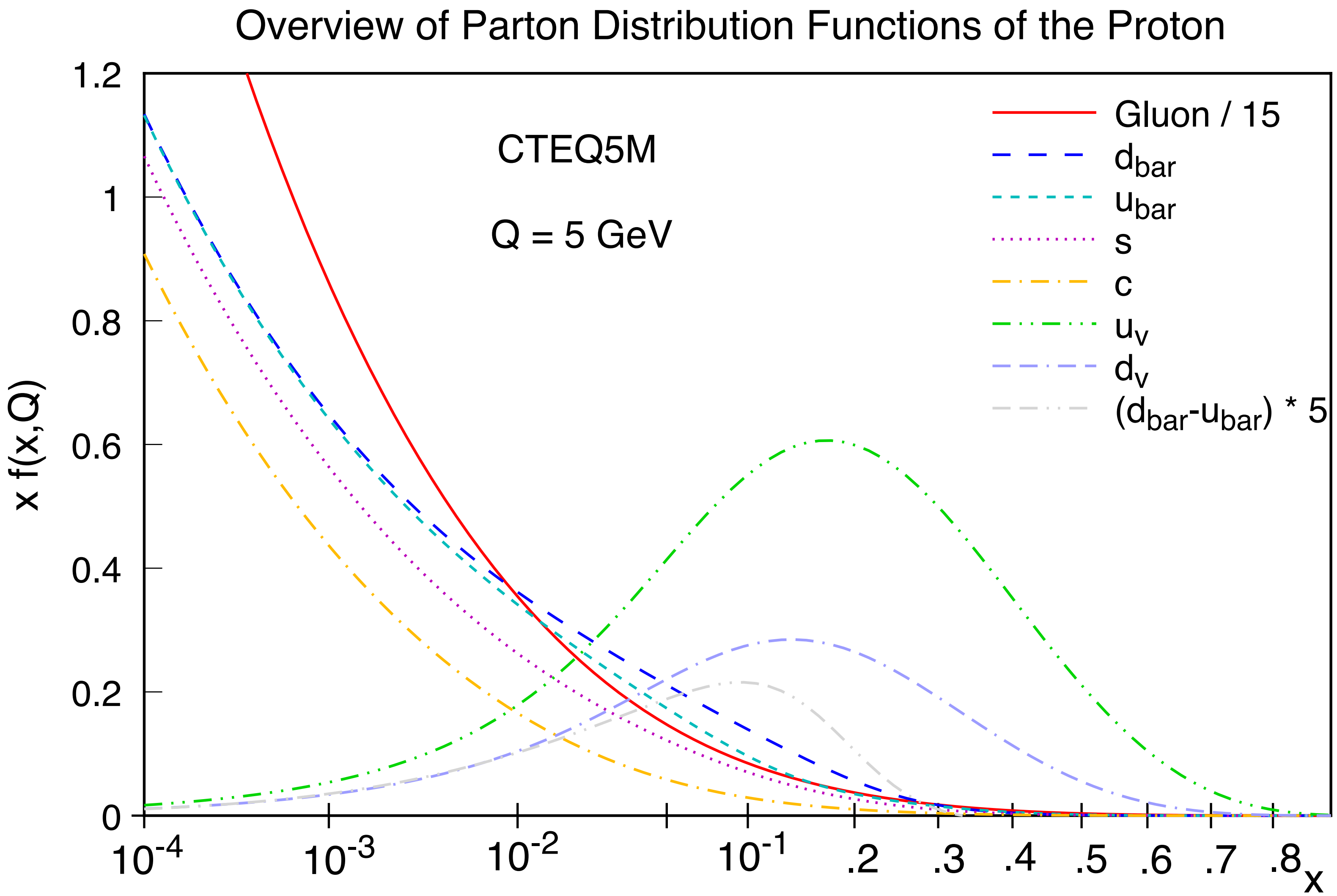




Note:

Strong interplay between possible new physics effects at large E_T and extraction of the gluon

Typical behaviour of parton densities in the proton



- All densities vanish as $x \to 1$
- At $x \to 0$
  - Valence quarks vanish
  - Strong rise of the gluon, which becomes dominant
  - Also sea quarks increase

➡ driven by the gluon through $g \to q\bar{q}$