Advanced Topics in Quantum Information Theory Exercise 5

FS 12
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Consider a non-equilibrium system of fermionic modes c_j in a 1D lattice ("wire"), whose dynamics is governed by

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -i[H,\rho] + \kappa \sum_{j=1}^{N-1} \left(d_j \rho d_j^\dagger - \frac{1}{2}\{d_j^\dagger d_j, \rho\} \right). \tag{1}$$

We will assume that

1. $H = 0 \rightarrow$ there is only dissipative dynamics.
2. $d_j = \frac{1}{2}\left(c_j + c_j^\dagger - c_{j+1} + c_{j+1}^\dagger\right)$ for $j = 1, \ldots, N-1$.

a) Show that the state

$$|\Psi_g\rangle = \frac{1}{\sqrt{N}} \prod_k \left(1 + \varphi(k) c_k^\dagger c_{-k}^\dagger\right) |0\rangle$$

is a steady state of (1). Here, $c_j = \sum_k e^{-ikx_j} c_k$ and $\varphi(k) = \cot\left(\frac{k}{2}\right) e^{ik}$.

b) Express $|\Psi_g\rangle$ and d_j in the Majorana basis.

c) Show that $|\tilde{\Psi}\rangle := (\gamma_{A,1} + i\gamma_{B,N})^\dagger |\Psi_g\rangle$ is also a steady state of (1), confirming that the dark subspace is spanned by $|\Psi_g\rangle$ and $|\tilde{\Psi}\rangle$.

Hint: Have a look at the article S. Diehl, E. Rico, M.A. Baranov, and P. Zoller, *Topology by dissipation in atomic quantum wires*, Nature Physics **7** (2011), 971–977.