Advanced Topics in Quantum Information Theory
Exercise 3

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Exercise 3.1 The Shor code and Stabilizers

We have seen in the previous exercise that the Shor code is useful for encoding a single qubit in 9 qubits. Now we will look at the Shor code in the stabilizer picture. The generators for stabilizer group for the Shor code has elements

g_1	Z_1Z_2
g_2	Z_2Z_3
g_3	Z_4Z_5
g_4	Z_5Z_6
g_5	Z_7Z_8
g_6	Z_8Z_9
g_7	$X_1X_2X_3X_4X_5X_6$
g_8	$X_4X_5X_6X_7X_8X_9$
$\bar{Z}$	$X^{\otimes 9}$
$\bar{X}$	$Z^{\otimes 9}$

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where we also define two Pauli group elements (that are not generators) $\bar{Z}$ and $\bar{X}$.

a.) Show that the generators stabilize the codewords

$$|0_L\rangle = \frac{1}{2\sqrt{2}}\left(|000\rangle + |111\rangle\right) \otimes \left(|000\rangle + |111\rangle\right) \otimes \left(|000\rangle + |111\rangle\right)$$
$$|1_L\rangle = \frac{1}{2\sqrt{2}}\left(|000\rangle - |111\rangle\right) \otimes \left(|000\rangle - |111\rangle\right) \otimes \left(|000\rangle - |111\rangle\right).$$

b.) Show that the operators $\bar{Z}$ and $\bar{X}$ act as logical Z and X operators on the logical bits $|0_L\rangle$ and $|1_L\rangle$. Show that $\bar{Z}$ and $\bar{X}$ are independent of and commute with the generators of the Shor code. Also show that $\bar{Z}$ and $\bar{X}$ anti-commute.

c.) Prove that any error X_i, Z_i, and X_iZ_i can be corrected by the Shor Code, where the position of the error, i, is arbitrary.

d.) Prove that two qubit errors of the form X_iX_j can also be corrected, but Z_iZ_j errors cannot ($i \neq j$).