Solid State Theory
Exercise 8

Exercise 8.1 Bohr-van-Leeuwen-Theorem

Prove the Bohr-van-Leeuwen-theorem, which states that there is no diamagnetism in classical physics.
Hint: $\mathcal{H}(p_1, \ldots, p_N; q_1, \ldots, q_N)$ is the Hamiltonian of the N-particle system with vanishing external magnetic field. In comparison, the Hamiltonian with applied magnetic field B is then given by $\mathcal{H}(p_1 - e/cA_1, \ldots, p_N - e/cA_N; q_1, \ldots, q_N)$, where $B = \nabla \times A$ and $A_i = A(q_i)$. The magnetization can be calculated using

$$M = \left\langle -\frac{\partial \mathcal{H}}{\partial B} \right\rangle = \frac{1}{\beta} \frac{\partial \log Z}{\partial B}, \tag{1}$$

with the partition function Z of the system in the magnetic field.

Exercise 8.2 Landau Diamagnetism

Calculate the orbital part of the magnetization of the free electron gas in 3D in the limits of low temperature and small external field ($T \to 0,\ H \to 0$). In addition, show that the magnetic susceptibility at $T = 0$ and $H = 0$ is given by

$$\chi = -\frac{1}{3} \frac{m^2}{m^{*2}} \chi_P, \tag{2}$$

where χ_P is the Pauli (spin-)susceptibility.
Hint: Calculate the free energy,

$$F = N\mu - k_B T \sum_i \ln \left[1 + e^{-(\epsilon_i - \mu)/k_B T} \right], \tag{3}$$

at $T = 0$ to second order in H using the Euler-Maclaurin formula,

$$\sum_0^{n_0} f(n) = \int_{-1/2}^{n_0+1/2} f(n) \mathrm{d}n - \frac{1}{24} \left[f'(n_0 + 1/2) - f'(-1/2) \right]. \tag{4}$$

Exercise 8.3 Landau Levels in Graphene

Graphene is defined as a single two-dimensional layer of graphite, the C-atoms are arranged on a two-dimensional honeycomb lattice (cf. exercise 5). The latter is not a Bravais-lattice, but a triangular lattice with a diatomic basis. Consequently, the reciprocal lattice, which is a honeycomb lattice as well, has two inequivalent points called K- and K'-points (see Fig. 1). The two atoms per unit cell create a valence- and a conduction band which cross linearly in one point (called the Dirac point) at the K- and K'-points and form the so-called Dirac cones (see Fig. 1). In undoped graphene, the Fermi energy is exactly at the Dirac point.
To a good approximation, the spectrum in graphene is linear at the Fermi energy and described by the Hamiltonian

$$\mathcal{H} = v_F (p_x \sigma_x \chi_0 + p_y \sigma_y \chi_z). \tag{5}$$

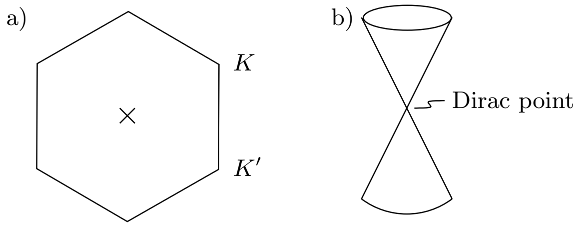

Figure 1: a) First Brillouin zone of graphene with K- and K'-points. b) Band structure of graphene at the K- and K'-points: the Dirac cones.

Here, the Pauli matrices σ act on a pseudo-spin and the Pauli matrices χ refer to the inequivalent points K and K'. Subsequently, it is enough to consider only one of the K- or K'-points; i.e., consider only $\mathcal{H} = v_F(p_x\sigma_x + p_y\sigma_y)$.

a) Using the Peierls-substitution $\boldsymbol{p} \to \boldsymbol{p} - (e/c)\boldsymbol{A}$, find the Landau levels in graphene for a magnetic field perpendicular to the plane (ignore the Zeeman-term).

   Hint: Take the "square" of the Schrödinger equation. Note that not only the σ_μ have non-trivial (anti-) commutation relations but also $\boldsymbol{p}$ and $\boldsymbol{x}$ do not commute.

b) Determine the degeneracy of the Landau levels.

c) Will the magnetization of graphene oscillate when changing the magnetic field? What is the dependence of the ground state energy and the magnetization on a small magnetic field?

Office hour:
Monday, April 18th, 2011 - 9:00 to 11:00 am
HIT K 11.3
Daniel Müller