Particle Physics Phenomenology II

FS 11, Series 4

Due date: 21.03.2011, 1 pm

Exercise 1 Electroweak charges and currents

Define the left handed SU(2) isospin doublet $\chi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$.

The weak current may then be expressed as $j_i^\mu = \bar{\chi}_L \gamma^\mu \tau_i \chi_L$ where the τ_i shall denote Pauli matrices.

i) Show that the current is conserved, i.e. $\partial_\mu j_i^\mu = 0$.
Hint: You may just assume a globally invariant free massless SU(2) doublet χ_L which satisfies Diracs equation $\not{\partial}\chi_L = 0$.

ii) Show that the conserved charge related to this current is given by

$$Q_i = \int d^3x \chi_L^\dagger \tau_i \chi.$$

iii) Further show that these charges satisfy

$$[Q_i, Q_j] = i\epsilon_{ijk} Q_k$$

and that they are therefore generators of SU(2).
Hint: Use the equal time anti-commutation relations of χ_L,

$$\{\chi_L^\dagger(\mathbf{x},t)_l, \chi_L(\mathbf{y},t)_k\} = \delta_{lk}\delta^3(\mathbf{x}-\mathbf{y}), \quad \{\chi_L(\mathbf{x},t)_l, \chi_L(\mathbf{y},t)_k\} = 0 = \{\chi_L^\dagger(\mathbf{x},t)_l, \chi_L^\dagger(\mathbf{y},t)_k\}.$$

iv) Compute the conserved charges corresponding to

$$j_\pm^\mu = \bar{\chi}_L \gamma^\mu \tau_\pm \chi_L, \quad j_3^\mu = \bar{\chi}_L \gamma^\mu \tau_3 \chi_L.$$

where $\tau^\pm = \tau_1 \pm i\tau_2$. Do their charges also generate SU(2)?

Exercise 2 Higgs couplings in the standard model

The Higgs part of the Standard model Lagrange density may be written as

$$\mathcal{L} = (D_\mu \phi)^+ (D^\mu \phi) - V(\phi)$$

where

$$D_\mu = \partial_\mu - igT^a W_\mu^a - ig' \frac{Y}{2} B_\mu, \quad V(\phi) = \mu^2 \phi^2 - \frac{\lambda}{4}\phi^4.$$

The electroweak symmetry is broken by expanding the Higgs field around its vacuum expectation value v, i.e. let

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}.$$

In what follows You will explicitely work out the gauge boson mass terms and the hWW,$hhWW$, hZZ and $hhZZ$ interaction terms.

i) You may start by substituting $Y = 1$, $T^a = \frac{1}{2}\tau^a$ and the explicit pauli matrices into the kinetic term.

ii) Diagonalise the quadratic terms by introducing the physical fields

$$W_\mu^+ = \frac{1}{\sqrt{2}} \left(W_\mu^1 - iW_\mu^2\right) = (W_\mu^-)^\dagger \tag{1}$$

$$Z_\mu = \frac{gW_\mu^3 - g'B_\mu}{\sqrt{g^2 + g'^2}} \tag{2}$$

$$A_\mu = \frac{g'W_\mu^3 + gB_\mu}{\sqrt{g^2 + g'^2}}. \tag{3}$$

iii) Identify the coefficients of mass and interaction terms in the following expansion

$$\begin{aligned}(D_\mu\phi)^\dagger(D_\mu\phi) &= (\partial_\mu h)^2 + M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu - iV_{hWW} hW_\mu^+ W^{-\mu} \\ &\quad -iV_{hhWW} hhW_\mu^+ W^{-\mu} - iV_{hZZ} hZ_\mu Z^\mu - iV_{hhZZ} hhZ_\mu Z^\mu.\end{aligned}$$

and “derive“ the corresponding Feynman rules.